MATHEMATICAL CALENDAR
2021-2022

Societat d'Educació Matemàtica Comunitat Valenciana Al-Khwarizmi

The Mathematical Education Society of the Valencian Community Al-Khwarizmi is a society of teachers of Mathematics. The objectives of the Company are, in accordance with its statutes:

1. Disseminate mathematics and the various currents of mathematical thought.
2. Transmit educational innovations in the teaching and learning of mathematics.
3. Promote the development and dissemination of research in Mathematics Education.
4. Encourage all those activities aimed at overcoming obstacles to the dissemination of mathematics generated by cultural or gender reasons.
5. Collaborate and exchange information with Associations and Societies of a similar nature and purpose.
6. Collaborate with institutions and entities to carry out studies and activities related to Mathematics and Mathematics Education.
7. Carry out studies, critiques and curricular proposals for any of the educational levels.

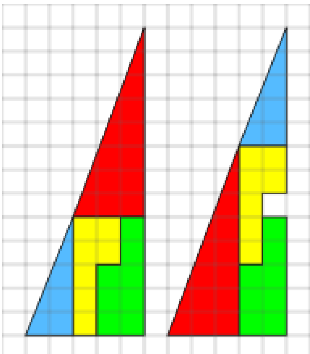
If you consider that these objectives are important, please contact us on the page

http://www.semcv.org/.

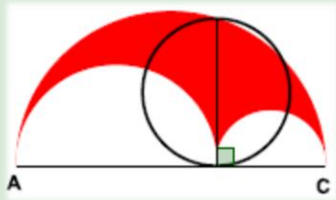
DIPUTACIÓ DE CASTELLÓ

SEPTEMBER

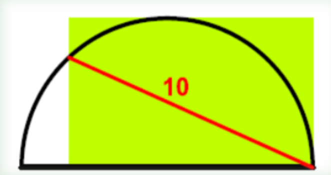
MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.



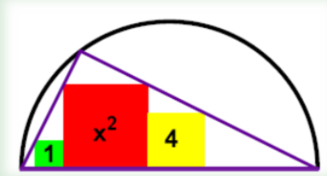
1 Three semicircles with centres in segment AC. Relate the area of the red zone and the area of the black circle



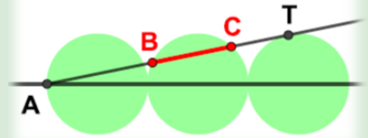
2 Semicircle, string of length 10 and rectangle. Find the area of the rectangle



3 Semicircle, three squares of areas 1, x^2 and 4 and triangle. Find x



4 Three equal circles, of radius 1, tangent to each other, T point of tangency. Find BC



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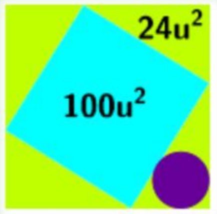
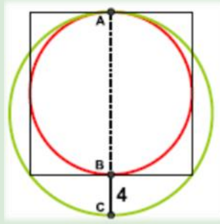
6 Two squares and a circle. Find the area of the circle

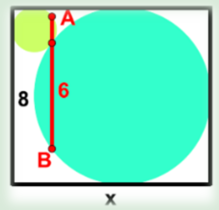


7 Two semicircles with diameters at AB || base. If the fuchsia line measures π, find the perimeter of the rectangle

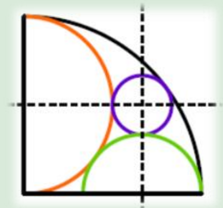
8 Two circles with centres at AB. If BC = 4 cm, find the area enclosed between the circles



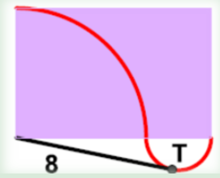
9 Two tangent circles and a rectangle of base x and height 8. If AB = 6. find x



10 A quadrant, a circle and two semicircles, all tangent to each other. Find the relationship between radii

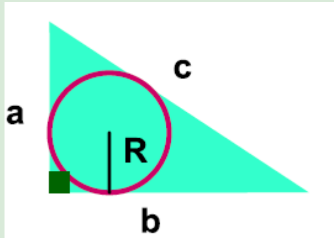


11 A quadrant, a semicircle, a rectangle, T is a point of tangency. Find the area of the rectangle

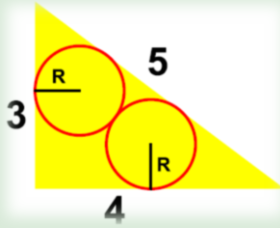


12

13 Find R as a function of a, b, and c

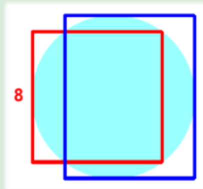


14 Find R



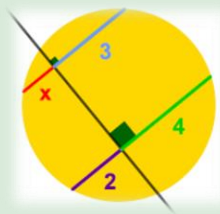
15 A square and a circle. Which of the two has the larger perimeter?

16 A square with side 8, a rectangle, and a circle. Find the area of the circle and the rectangle



17 A right triangle with hypotenuse 13. Inscribed circle of radius 2. Find the area of the triangle

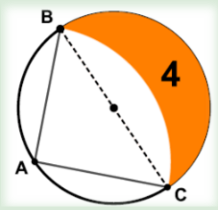
18 Circle and three strings, two of them parallel. Find x



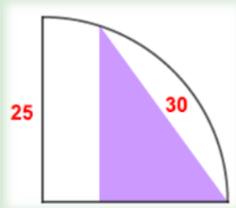
19

20 A quadrant and a right triangle of legs 3 and 4. Find the area of the quadrant

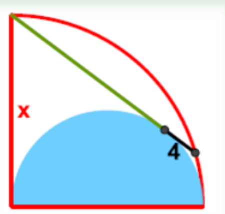
21 A circle of diameter BC and a quadrant. Area of the shaded area 4. Find the area of the circle



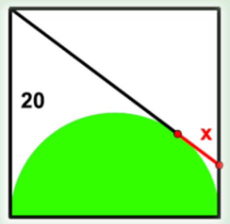
22 Quadrant with radius 25 and right triangle of hypotenuse 30. Find the legs of the triangle



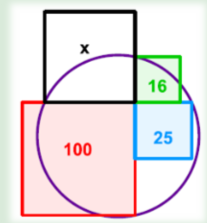
23 A quadrant, a semicircle, and a tangent chord. Find x



24 Square with side 20, semicircle and tangent chord. Find x



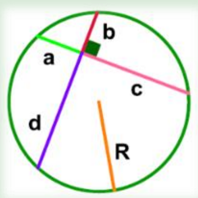
25 Four squares of areas 100, 25, 16, and x. Find x and the area of the circle



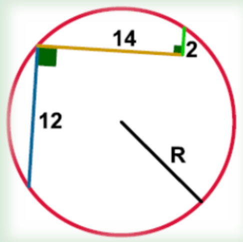
26

27 FAURE'S THEOREM: In a circle of radius R and two perpendicular chords:

$$a^2 + b^2 + c^2 + d^2 = 4R^2$$

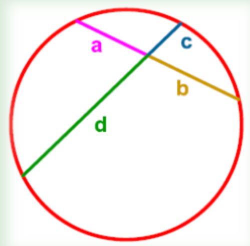


28 Find the radius of the circumference



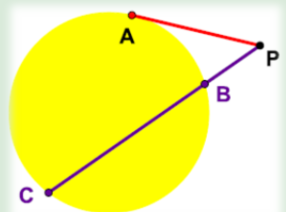
29 STRING THEOREM:

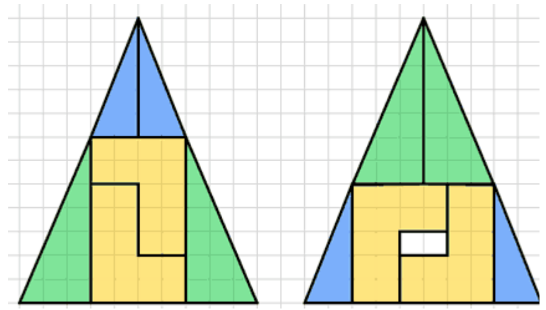
$a \cdot b = c \cdot d$



30 POWER OF A POINT: Circle, secant and tangent. Prove that:

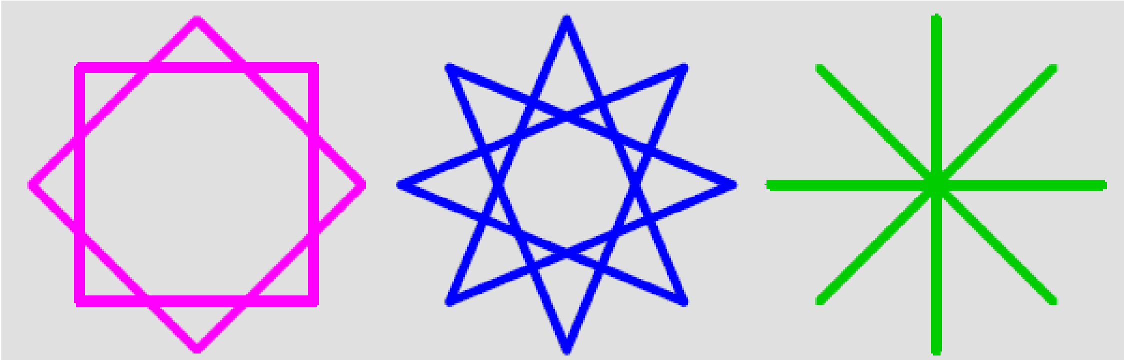
$PA^2 = PB \cdot PC$

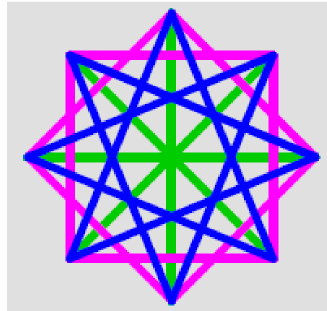




OCTOBER

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.





1

2

1.- If $\mathbf{x} = \left(1+\frac{1}{n}\right)^{n}$ and $\mathbf{y} = \left(1+\frac{1}{n}\right)^{n+1}$ compare $\mathbf{x^y}$ whit $\mathbf{y^x}$

2.- Prove that, for every positive integer n: $\mathbf{1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^n \cdot (n-1)^2 + (-1)^{n+1} \cdot n^2 = (-1)^{n+1} \cdot (1 + 2 + \cdots + n)}$

3

4 **5**

Let n be a fixed positive integer. To any choice of n real numbers that satisfy $0 \le x_i \le 1$ ($i \in \{1, 2, .., n\}$) we make them correspond the sum

$$\sum_{1 \le i < j \le n} |x_i - x_j| = |x_1 - x_2| + |x_1 - x_3| + \cdots + |x_1 - x_n| + |x_2 - x_3| + \ldots + |x_2 - x_n| + \ldots + |x_{n-1} - x_n|$$

Find the largest possible value of this sum

6

Simplify:

$$\sqrt[3]{\frac{1\cdot 2\cdot 4 + 2\cdot 4\cdot 8 + \cdots + n\cdot 2n\cdot 4n}{1\cdot 3\cdot 9 + 2\cdot 6\cdot 18 + \cdots + n\cdot 3n\cdot 9n}}$$

7

8

A function y = f (x) is said to be periodic if there exists a positive real number p such that f (x + p) = f (x) for all x. For example, y = sin x is periodic of period 2π. Is the function:

$$\mathbf{y = \sin (x^2)}$$

periodic? Prove your claim

9

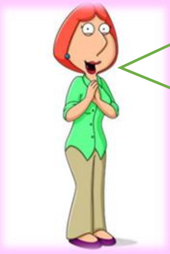
A sequence $a_1, a_2, a_3, \ldots$ satisfies that $a_1 = ½$ and $a_1 + a_2 + \cdots + a_n = n^2 \cdot a_n$, for any n. Determine the value of a_n

10

11

Let A, B, C and D be four consecutive points of a circle and P, Q, R and S the midpoints of the arcs AB, BC, CD and DA. Prove that PR ⊥ QS

12



13

For each real r, we define:

$$[r] = \max\{z \in \mathbb{Z} \mid z \le r\}$$

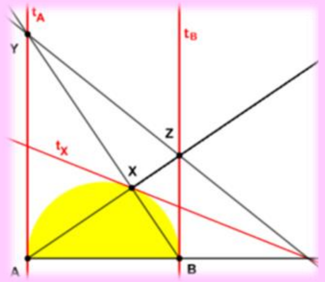
(e. g. [6] = 6; [π] = 3; [-1,5] = -2)). Draw the set of points on the (x, y) plane:

$$\mathbf{[x]^2 + [y]^2 = 4}$$

14

Given four weights that are in arithmetic progression and a two-arm scale, show how to find the largest weight using the scale only twice.

15



16

Given a circle of diameter AB and a point X different from A and B, let t_A, t_B and t_X be the tangents to the circle at A, B and X. Let Z be the intersection of AX with t_B and Y the intersection of BX with t_A. Prove that the three lines AB, t_X and ZY are concurrent or parallel

17

18 **19**

We have an unlimited number of 8-cent and 15-cent stamps. Some amounts of postage cannot be obtained exactly, e. g. 7 cents or 29 cents. What is the largest quantity that cannot be obtained exactly, i. e. the amount of postage that cannot be reached exactly, while all higher amounts are achievable?

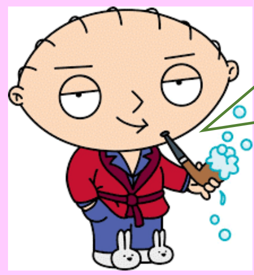
20

Let AB be one of its diameters of a given circle. Let C be a fixed point on segment AB and Q be a variable point on the circumference of the circle. Let P be a point on the line determined by Q and C for which:

$$AC/CB = QC/CP$$

Describe the locus of point P

21



22

Show that if a number is rational, the decimal part, the integer part, and the number cannot be in geometric progression.

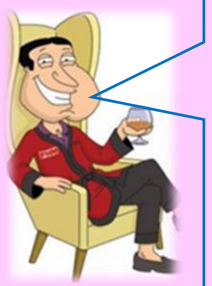
Find a positive number such that its decimal part, its integer part and the number are in geometric progression

23

Let n be a positive integer. Prove that if n is a power of 2 then n cannot be put as the sum of consecutive positive integers

24

25



26

Suppose:

$$\mathbf{n\cdot(n+1)\cdot a_{n+1} = n\cdot(n-1)\cdot a_n - (n-2)\cdot a_{n-1}}$$

for every positive integer n ≥ 1. Si $a_0 = 1$ and $a_1 = 2$, find:

$$\sum_{i=0}^{50} \frac{a_i}{a_{i+1}} = \frac{a_0}{a_1} + \frac{a_1}{a_2} + \cdots + \frac{a_{50}}{a_{51}}$$

27

28

Let ABCD be a rectangle with BC = 3 · AB. Prove that if P and Q are points on BC with BP = PQ = QC, then:

$$\angle DBC + \angle DPC = \angle DQC$$

29

30

Two seventh grade students were allowed to play in an eighth grade chess tournament. Each pair of participants played each other once and each of the participants received one point for winning each game, half for drawing a draw, and zero points for each game lost. The two seventh graders received a total of eight points, and the eighth graders all earned the same number of points. How many eighth grade students participated in the tournament? Is it the only solution?

31

(http://cms.math.ca/Competitions/CMO/) ORGANIZES: Rafael Martínez Calafat. Retired teacher

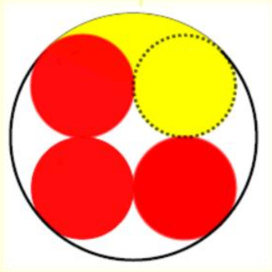
NOVEMBER

MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY | SUN.

1

The figure shows four circles of radius 1 inscribed in a larger circle.
What is the area of the yellow zone?

2

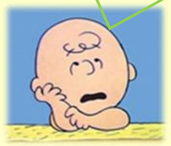


3

Three friends decide to sell orange granite at the entrance of a sports stadium. They bought oranges and sugar and paid € 50. In addition, they paid € 100 for the rental of tables, purchase of glasses and drinking straws. They calculated to get 250 glasses of granite. How much should they sell each glass for to make a 25% profit?

4

From the top of a lighthouse located 127.5 m above sea level, you can see the horizon, how far is it from the lighthouse, knowing that a round the world is 40,000 km?



5

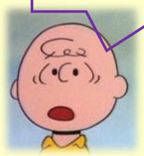
6

We have 12 50-cent euro coins and a one-euro coin. We put them in a circle. Starting with the coin you want, you have to count 13 coins to the right and the one in that place is eliminated. We go back to counting 13 coins starting with the next one, to our right, from which we have just withdrawn. We repeat this operation until we only have one coin. By which currency should we start counting so that the last one we withdraw is the one-euro coin?

7

8

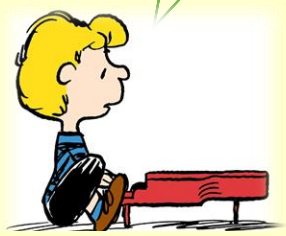
What is the natural minor that divided by 2, 3, 4, 5, and 6 gives the remainder 1, 2, 3, 4, and 5, respectively?



9

Find the naturals so that all their divisors, except 1, are even.

10



11

Four wounded soldiers have to cross a seriously damaged bridge at night to escape the enemy. The bridge only supports the weight of two soldiers at a time. When two soldiers cross it, they must do so at the speed of the slowest. The four soldiers only have one flashlight that has to be used every time they cross the bridge.
Individually they take 1, 2, 4 and 6 minutes to cross the bridge. What is the minimum time it takes for the four of them to cross it?

12

Participants in a television contest must answer 30 questions. In each question if they do well, they get a point. If they fail, half a point is subtracted. If they do not respond, they receive zero points. If a contestant received six points, detail her responses.

13

One person has a saddle valued at € 50 and two horses. If you place the chair in the first, its value is twice the second. If you place the chair in the second, its value is triple the first. What is the value of each horse?

14

15

Each of two vertical poles of different heights located on a flat ground, has a device, in its upper part, that directs a laser beam to the base of the other pole. If the rays cross at a height of 24 m from the ground and if the shortest of the poles has a height of 40 m, what is the height of the tallest pole?

16

17

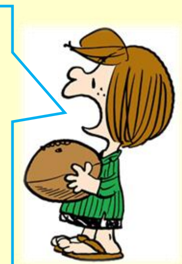
I have two coins. One has a 7 on one side and the other has a 10. If we toss the two coins into the air and add the results that appear on their upper sides, we obtain: 11, 12, 16 and 17. What numbers can be the numbers of the two coins?

18

19

Two elevators leave from the sixth floor of a building at two in the afternoon and both are going down. The fastest takes a minute to go from one floor to another, while the slowest takes two minutes. The first elevator to reach a floor will have to stop for three minutes to open doors, raise and lower passengers, and close doors. Which elevator gets to the first floor first? At what time will each elevator reach the ground floor?

20



21

22

23

Find the naturals such that half of their divisors are even and the other half are odd

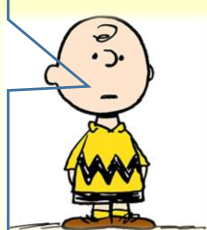
24

Find the pairs of four-digit palindromes whose sum is a five-digit palindrome

25

At home I have an alarm clock that goes back two minutes every hour, while my wristwatch advances one minute every hour. One day I set both clocks to correct times and left the house. When I went back into the house, in the clock on my wrist was 12 at night and, on the other hand, on the alarm clock it was 11 at night. How long was I away from home? At what exact time did you enter the house?

26



27

The product of a two-digit number and its own numbers is 1950. Find this number

28

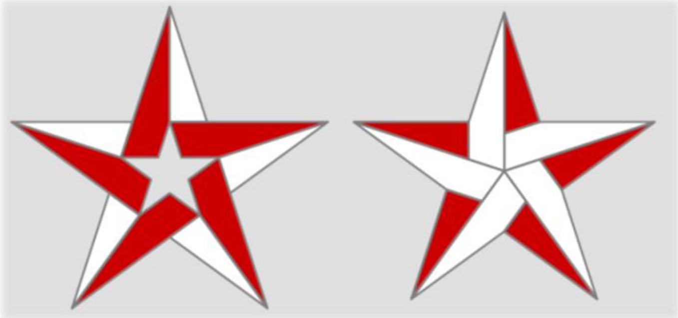
29

Place 1, or –1, in each box on a 4x4 grid so that the product of all the numbers in a row or column is always –1. What is the minimum and maximum amount of –1 that we should put in? What would these quantities be on an nxn grid?

30

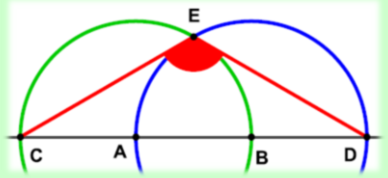
DECEMBER

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.



1 The sum of three digits gives 15. If one of them is replaced by 3, the product of the new digits gives 36. What digits were at the beginning?

2

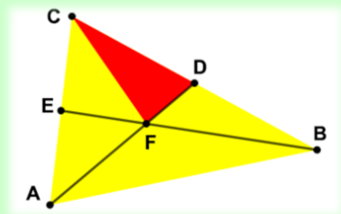


3 In the image there are two equal circles with centres A and B. Each of them passes through the centre of the other and the line that passes through A and B intersects the circles at C and D. If E is the intersection of the two circles, find ∠CED

4 On a math test, if each boy had scored three more points than he did scored, the class average would have been 1.2 points higher than it was. Find the percentage of girls in the class

5

6

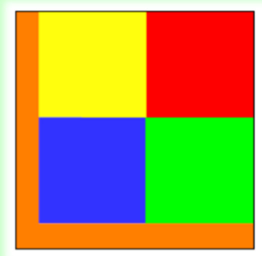


7 Given the triangle ΔABC, let BE and AD be two medians whose intersection is F. Suppose $A_{\Delta FDC} = 3$. Find the area of the triangles ΔEAB and ΔAFB and the area of the quadrilateral EFDC.

8 Find the naturals less than 100 with the greatest number of divisors

9 I have two dice, one red and one blue. If I roll both at the same time, what is the probability that the number on the red die is greater than the number on the blue die?

10

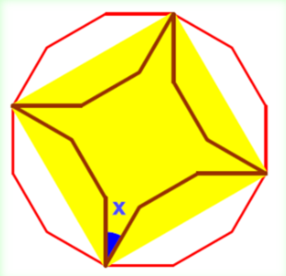


11 We divide a square of 125 cm² in area into five regions, four squares and an L-shaped polygon, all of equal area. What is the length, in cm, of the shortest side of the L-shaped polygon?

12

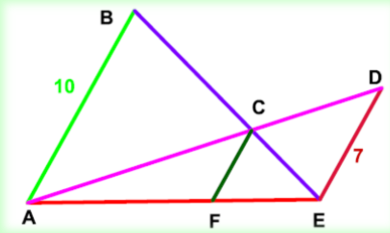
13 Find the naturals whose square and the number itself end in the same two digits and in the same order

14



15 In a regular dodecagon we have inscribed a square, as shown in the figure. In addition, we have drawn the symmetrics of the sides of the dodecagon with axis of symmetry the sides of the square. Find the measure of angle x and the area of the star if the side of the dodecagon is 1

16

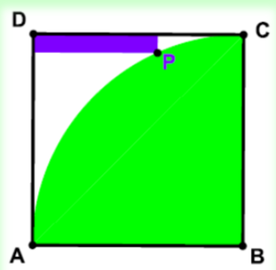


17 In the attached figure the segments AB, CF and ED are parallel. If the length of AB is 10 and the length of ED is 7, find the length of the segment CF

18 With the digits 1, 2, 3, 4 and 5 written in some order we form the PQRST number. If PQR is a multiple of 4, QRS is a multiple of 5 and RST is a multiple of 3, find the number PQRST

19

20

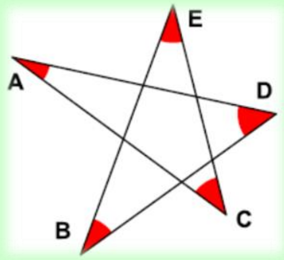


21 There is a square ABCD and a quadrant of radius CB and centre B. P is a point of the quadrant that is eight units from the DA side and one unit from the DC side. Find the side of the square

22 It is known from the natural N that it is a multiple of p but it is not a multiple of 2p. Find the remainder of N by dividing by 2p

23 We inscribe a semicircle in an isosceles triangle of base 16 and height 15, as shown in the figure. Find the radius of the semicircle

24



25 What is the sum of the measure of the angles A, B, C, D and E of the star in the attached figure?

26

27 Two sides of a quadrilateral are 4 and 1. One of the diagonals, of length 2, divides the quadrilateral into two isosceles triangles. Find the perimeter of the quadrilateral

28

2	4		2
	3	3	
6		1	

29 Fill cells in the attached matrix with digits so that all rows add up to the same, all columns add up to the same, even though the sum of a row may be different from the sum of a column

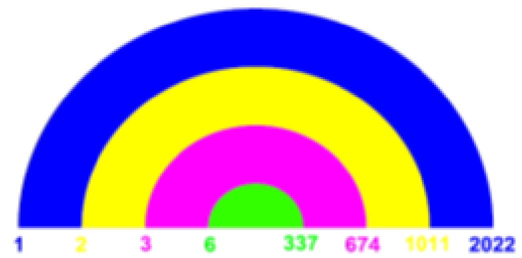
30 Calculate the remainder of dividing

$$x^{100} - 2x^{99} + 4$$

by

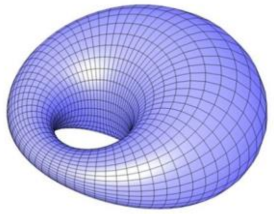
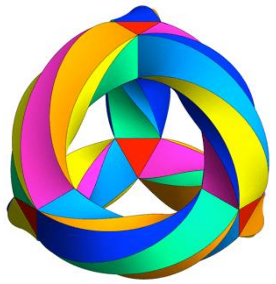
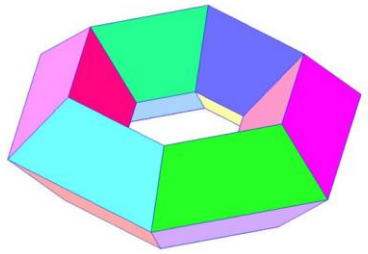
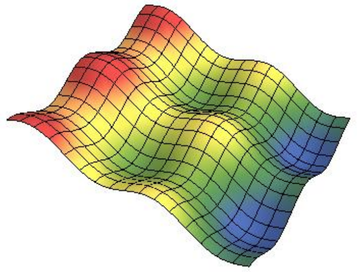
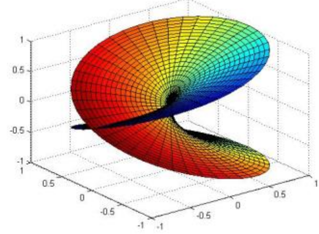
$$x^2 - 3x + 2$$

31 A bag contains 3 red and 2 green balls. We remove, one by one and without return, balls from the bag until we have removed all of the same colour. What is the probability that we have drawn the 3 red balls?



JANUARY

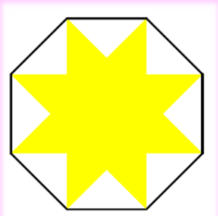
MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.

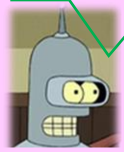
1 How many numbers less than 100 are the product of three prime numbers?

2

3 In the figure there is a regular octagon with side 4 cm. Find the area of the octagonal star



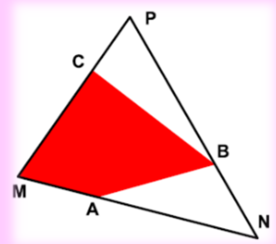
4 A bag contains m white and n black balls. We draw a ball at random and return it by adding k balls of the same colour as the one drawn. We draw another ball at random, what is the probability that the second ball drawn is white?



5 Calculate the sum of all the products of two different naturals taken from 1 to n.

6 **7**

Points A, B, and C in the figure divide each side of the triangle ΔMNP into two pieces that are in the ratio 1: 3. Find the fraction of the area of the triangle ΔMNP coloured red



8 For what values of x is the expression:

$$\frac{\sin^3 x \cdot \cos x}{1 + \mathrm{tag}^2 x}$$

reaches its highest value and what is this?

9

10 If

$$x^2 + x \cdot y + y^2 = 84$$
$$x - (x \cdot y)^{1/2} + y = 6$$

calculate x·y

11 The drawing shows a quadrant of radius s and two tangent semicircles. Find the radius of the small semicircle.

12 In a right triangle, the bisector of an acute angle cuts the opposite leg into two pieces of length 1 and 2: What is the length of the bisector segment inside the triangle?

13 Let us consider the naturals with nine figures. How many numbers do we have to extract to ensure that at least two of them have the same number in the tens of thousands?

14 Solve **f(f(f(x))) = 0**, being:

$$f(x) = \begin{cases} x + 4 \text{ sii } x \leq -2 \\ -x \text{ sii } -2 < x < 0 \\ x \text{ sii } x \geq 0 \end{cases}$$

15 From the function f (x) it is known that it is periodic of period 5 and that in [3, 8 [verifies:

$$f(x) = x^2 - 10x + 25$$

Find f(2022)

16

17 How many pairs of integers (x, y) with $x \leq y$, verify that their product is equal to five times their sum?

18 What is the remainder of the division of

$$P(x)=x^{200}-2x^{199}+x^2+x+1$$

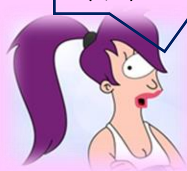
between $D(x)=x^2-3x+2$?

19 There are ten consecutive naturals. The sum of nine of them gives 2022. What number have we not added?

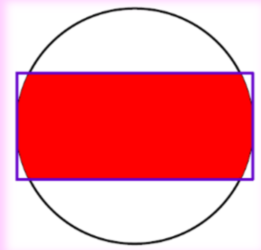
20 Points A and B are points on the graph of

$$y = x^2 - 7x - 1$$

Find the length of segment AB if (0, 0) is its midpoint



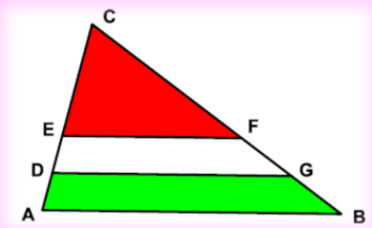
21 **22**



The circle and the rectangle in the figure have the same centre. The dimensions of the rectangle are 6x12 and the small sides of the rectangle are tangent to the circle, what is the area of the region common to the circle and the rectangle?

23

24/31 **25**

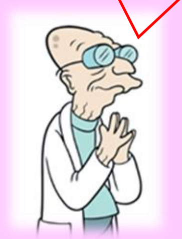


In the drawing EF // DG // AB. The shaded areas have the same area and CD = 4·DA

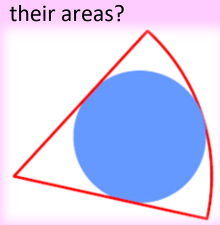
Find the ratio between CE and EA

26 Solve in ℝ

$$x^2 + y^2 = |x| + |y|$$

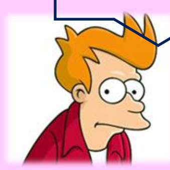


27 If the quotient between the radius of the circular sector and the radius of the circle is three, what is the quotient between their areas?

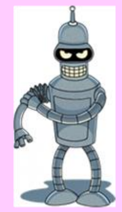


28 If the larger base of an isosceles trapezoid measures the same as the diagonal and the smaller base measures the same as the height of the trapezoid, find the ratio of the length of the smaller base to the length of the larger base.

29 A square has one vertex at the point P (1, 2) and another on the line y = 3x + 4. What is the smallest possible value for its area?

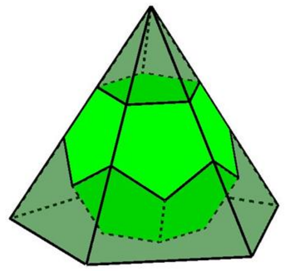


30

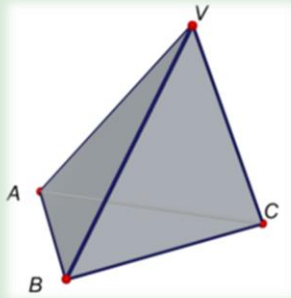


MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.

FEBRUARY



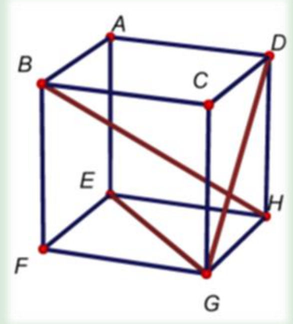
1



2

The base of a tetrahedron is an equilateral triangle, and the three lateral faces unfolded and laid in a plane form a trapezoid with sides 10, 10, 10, and 14 units in length.
Find the sum of the lengths of all the arista of the tetrahedron and also determine their area.
KöMaL C1559.

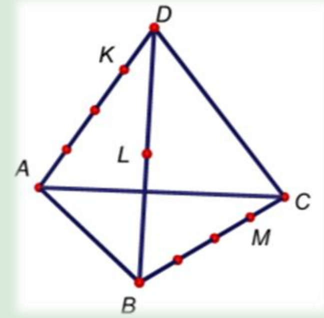
3



4

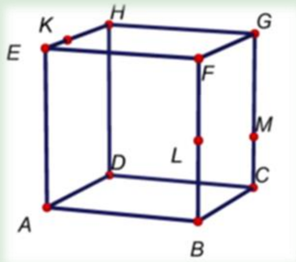
Let a cube ABCDEFGH, whit edge 1.
Prove that $\overline{BH}$ is perpendicular to $\overline{EG}$.
Prove that $\overline{BH}$ is perpendicular to $\overline{GD}$.
Prove that $\overline{BH}$ is perpendicular to plane EDG.
Calculate the intersection of $\overline{BH}$ and the plane EDG.
Calculate the distance of $\overline{BH}$ the plane EDG

5



6

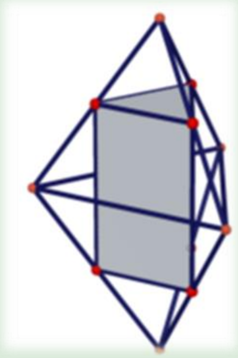
7 e-day



8

Let ABCDEFGH be a cube with edge $\overline{AB} = 1$. Let K of edge $\overline{EH}$ such that $\overline{HK} = 2 \cdot \overline{EK}$. Let L be the midpoint of edge $\overline{BF}$. Let M of the edge $\overline{CG}$ such that $\overline{GM} = 2 \cdot \overline{CM}$. Determine the sides of the section of the cube generated by the plane that passes through points K, L, M.

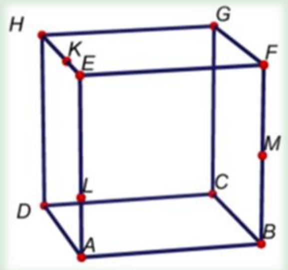
9



10

Let a cube ABCDEFGH of edge $\overline{AB} = 1$.
Let K of the edge $\overline{EH}$ such that $\overline{HK} = 2 \cdot \overline{EK}$
Let L of the edge $\overline{AE}$ such that $\overline{EL} = 2 \cdot \overline{AL}$
Let M the midpoint of the edge $\overline{BF}$.
Determine the perimeter and area of the section of the cube that determines the plane that passes through points K, L, M.

11



12

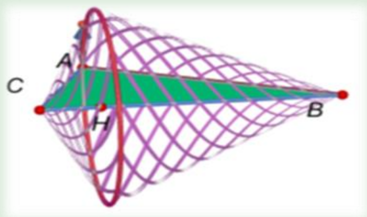
Let the regular tetrahedron ABCD of edge 1. Let K be the point of the edge $\overline{AD}$, such that $\overline{AK} = 3 \cdot \overline{DK}$. Let L be the midpoint of the edge $\overline{BD}$. Let M the point of the edge $\overline{BC}$ such that $\overline{BM} = 3 \cdot \overline{CM}$. Calculate the area of the tetrahedron section determined by the plane that passes through the points K, L, M

13

14

The hypotenuse of a right triangle is 5. Find the legs knowing that the volumes generated by the triangle as it rotates around the legs are one double that of the other. Find the volume of the two cones. Determine the volume of the double cone generated by the triangle when rotating about the hypotenuse

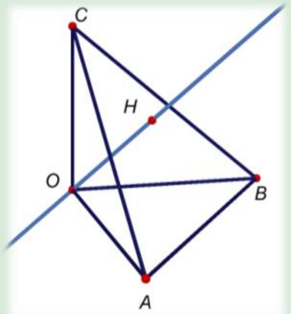
15



16

Two regular tetrahedron are joined by one face.
Determine the ratio between the volume of the vertex prism the midpoints of the edges of the tetrahedron and the sum of the volumes of the two tetrahedrons.

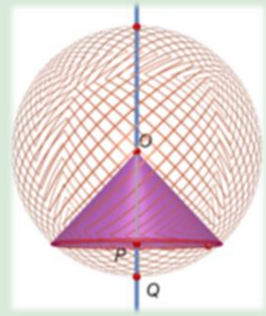
17



18

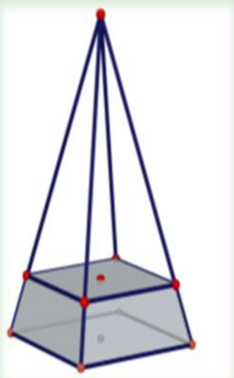
A sphere of radius r has inscribed a cone that has the vertex at the centre of the sphere and an angle 2α at the vertex.
Determine the area and volume of the part of the sphere that the cone intersects.
Problem proposed by Joan Galiana, student and mathematician

19



20

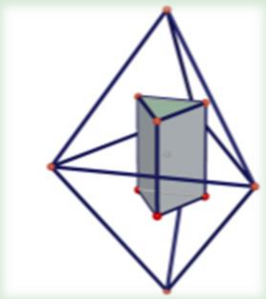
21



22

Given the regular double tetrahedron, determine the ratio between the volumes of the dual polyhedron (prism of vertices the centres of the 6 faces) and of the regular double tetrahedron

23



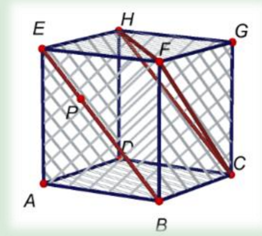
24

The edges emerging from vertex O of the tetrahedron OABC are perpendicular two by two. Show that the orthogonal projection H of O onto the face ΔABC is the orthocentre of the triangle ΔABC. Prove that:

$$\frac{1}{OH^2} = \frac{1}{OA^2} + \frac{1}{OB^2} + \frac{1}{OC^2}.$$

Show that the symmetric of O with respect to the centroid of the tetrahedron is the centre of the sphere circumscribed to the tetrahedron.

25



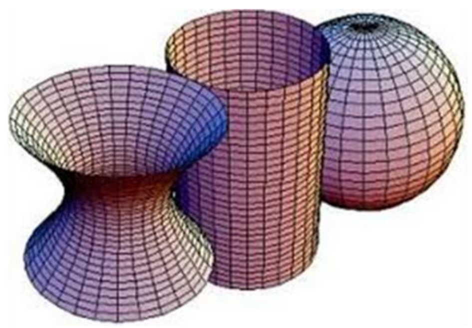
26

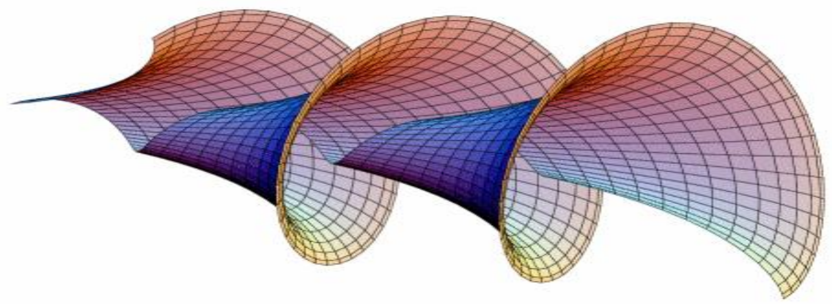
Let ABCDEFGH be a cub of edge 1.
Let P one point of the segment $\overline{BE}$ such that $\overline{EP} : \overline{BE} = 1:3$.
Find the distance from point P to the plane determined by the vertices C, F, H of the cube.

27

28

The height of a lateral face of a regular quadrangular pyramid is twice the edge of the base. What percentage of this height of the pyramid (counting from the base) do we have to cut with a plane parallel to the base so that the total area of the lateral surface plus the upper square of the resulting trunk of the pyramid is equal to half the lateral surface of the original pyramid.

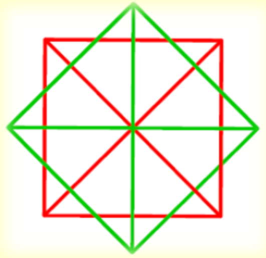




MARCH

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.

1

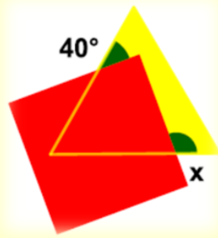


How many squares are in the figure? And triangles?

2

How many three-digit palindromes are multiples of three? And eleven?

3



In the figure we have a square and an equilateral triangle. Find x

4 5

Don Twisted has invented this game: He gives you a number, if it is even you multiply it by two and add one, if it is odd you multiply it by three and add one. If after applying the rule to the number that Mr. Twisted has given you and twice in a row to each of the numbers you are getting, you reach 208, what number did Don Twisted give you?

6

7 8

Grandfather Gerardo has distributed his collection of coins among his six grandchildren. He gave Carlos half of what he had. He gave Ferran half of what he had left. He gave Dani half of what he had left and so he continued first with Laia, then with Aitana and finally with Clara and kept three coins. How many coins did he have at the beginning and how many did he give to each grandchild?

9

The product of three different naturals is 30. What are the possible values of the sum of the three naturals?

10

$$\begin{array}{r} 7\;9\;5\;1\;6\;3 \\ -\;4\;9\;6\;7\;1\;8 \\ \hline \end{array}$$

Eliminate three digits in the top number and the bottom number so that the result of the new subtraction is the smallest possible

11

12

Dani and other partners have formed the AVANT club. At the parties, each member has invited as many people as his peña partners. If it is known that there will be more than 66 attendees and less than 99, how many people will attend the event?

13

14

How many triangles can we form that have their vertices at the vertices of a regular pentagon? And in a regular hexagon?

π day

15

Using the digits 8, 0, 7, 2, 6, 2, 5, 4 only once each, you have to generate four numbers with two digits less than 53 such that there are not two of them consecutive. Which are?

16

Place all the natural numbers from 1 to 9 without repeating any in the attached matrix, taking into account that the outer numbers indicate the product of the numbers located in the row or column

17

One meter is one billion nanometres. To calculate the thickness of a leaf, Lucia, has observed that ten leaves measure one millimetre. How many nanometres is the thickness of a sheet?

18

Dani has a sheet of dimensions 40 cmx20 cm. With three cuts, she divides this sheet into four equal rectangles. Each of these rectangles divides them into four equal ones, with the same type of cuts. This last operation she repeats twice. What is the perimeter of all the rectangles that are obtained at the end?

19

$$\begin{array}{r} 1\;A\;B\;C\;D\;E \\ \times\;\;3 \\ \hline A\;B\;C\;D\;E\;1 \end{array}$$

A, B, C, D, and E represent different digits. If the above product is well done, calculate the value of each letter

20

21

Calculate the possible values of A and B if the ¾ of the 2/5 of A is equal to the 2/3 of the 3/5 of B

22

Order from highest to lowest:

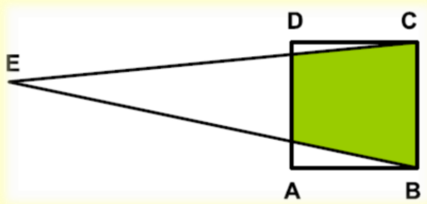
11^{525}, 1317^{175}, 37^{350}

23

			21
			60
			288
112	72	45	

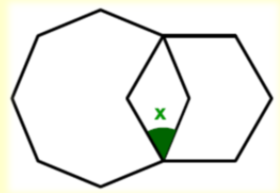
24 25

The area of square ABCD is 16 cm^2 and that of triangle ΔBCE is 32 cm^2. Find the area of the shaded trapezoid



26

In the figure there is a regular hexagon and regular octagon. Find the measure of angle x



27

28 29

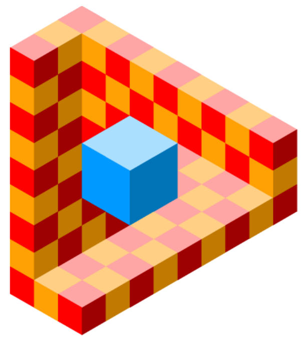
Aitana has written on a sheet of paper all the natural numbers that she can write. Laia has deleted those that, according to her, are prime numbers and has added them, obtaining 230. The older brother, Dani, congratulates Aitana because he has not forgotten any number and tells Laia that he has added a number that is not Prime number. Up to what number has Aitana written? What number has Laia considered a prime number and she is not?

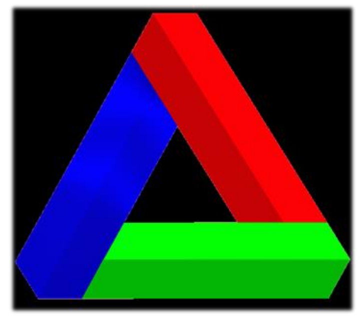
30

Dani collects geometric figures. Half of the ones he has are triangles, a third of the rest are circles, and a quarter of the ones that remain are trapezoids. If he has 20 trapezoids, how many triangles and circles does he have?

31

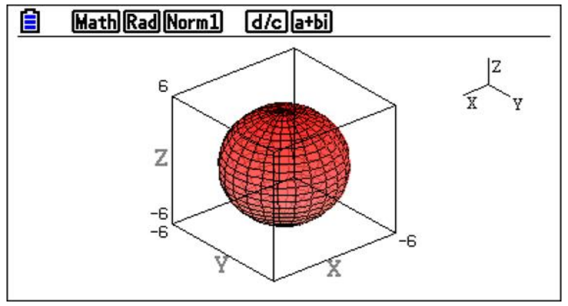
There are 200 people in a movie theatre. 130 of them are women. Also, there are 90 people who wear glasses. If half the men wear glasses, how many women don't?





APRIL

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.



1

Let the sphere be the equation:

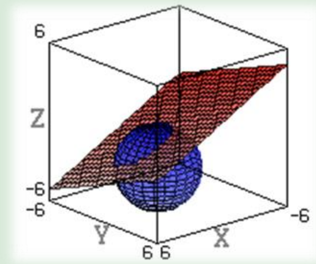
$E \equiv x^2 + y^2 + z^2 - 2x + 6z = 0.$

Determine the coordinates of the centre and the radius measure. Check if the plan:

$\Pi \equiv 3x - 2y + 6z + 1 = 0$

and the sphere are secant. Determine the radius of the intersecting circle of E, Π. Determine the centre of the intersection circle of E, Π.

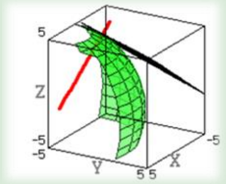
2



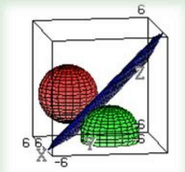
3

4 Find the equation of the sphere with centre C (3, -5, -2) tangent to the plane:

$2x - y - 3z + 11 = 0$



5 Find the equation of the sphere of radius 3, which is tangent to the plane $x + 2y + 2z + 3 = 0$ at the point A (1, 1, -3)



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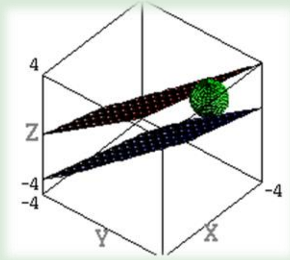


7

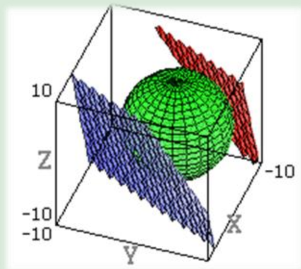
8

Determine the equation of the sphere that is tangent to the planes:

$\Pi \equiv 6x - 3y - 2z - 35 = 0$

$\Omega \equiv 6x - 3y - 2z + 63 = 0$

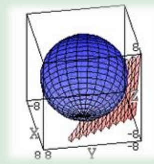
knowing that the point M (5, -1, -1) is a point of tangency in one of the planes.



9 Determine the equation of the plane tangent to the sphere

$x^2+y^2+z^2=49$

at point M (6, -3, -2)



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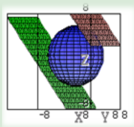
11



12

Determine the equations of the planes tangent to the sphere

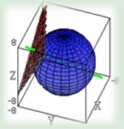
$(x-3)^2 + (y+2)^2 + (z-1)^2 = 25$

parallel to plane $4x + 3z - 17 = 0$

Show that the plane $2x - 6y + 3z - 49 = 0$ is tangent to the sphere

$x^2 + y^2 + z^2 = 49$

Calculate the coordinates of the point of tangency



13

A sphere has its centre on the line

$r \equiv \begin{cases} 2x + 4y - z - 7 = 0 \\ 4x + 5y + z - 14 = 0 \end{cases}$

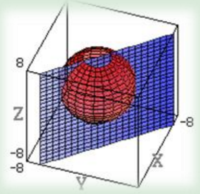
and is tangent to the planes

$\Pi \equiv x + 2y - 2z - 2 = 0$

$\Omega \equiv x + 2y - 2z + 4 = 0$

Determine its equation.

14 Determine the equation of the circumference that passes through the points A (3, -1, -2), B (1,1, -2) and C (-1,3,0)



15

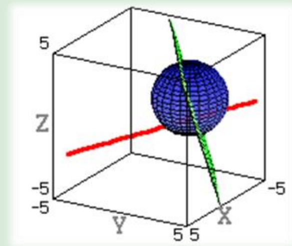
16

Determine the relative position of the line

$r \equiv \begin{cases} x = 2 - 2\alpha \\ y = -\frac{7}{2} + 3\alpha \\ z = -2 + \alpha \end{cases}$

and the sphere

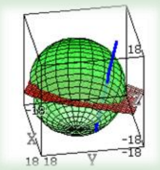
$E \equiv x^2 + y^2 + z^2 + x - 4y - 3z + \frac{1}{2} = 0$



17

18 Determine the equation of the sphere with centre O (2,3, -1) that intersects the line

$s \equiv \begin{cases} 5x - 4y + 3z + 20 = 0 \\ 3x - 4y + z - 8 = 0 \end{cases}$



with a chord of length equal to 16.

19

20

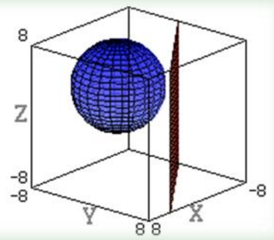
In the sphere of equation

$(x-1)^2 + (y+2)^2 + (x-3)^2 = 25$

determines the point M closest to the plane

$\Pi \equiv 3x - 4y + 19 = 0$

and calculate the distance from point M to this plane.



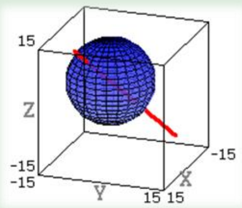
21

22

Find the shortest distance from point A (1, -1,3) to the sphere

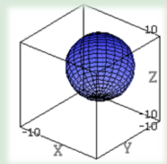
$E \equiv x^2 + y^2 + z^2 - 6x + 4y - 10z - 62 = 0$

At what point on the sphere is the shortest distance achieved?



23 Determine the equation of the sphere that passes through the points A (3,1, -3); B (-2,4,1); C (-5,0,0) and has the center in the plane:

$\Pi \equiv 2x + y - z + 3 = 0$



24

25

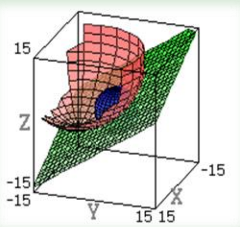
26

Let the spheres be equations:

$E_1 \equiv x^2 + y^2 + z^2 = 25$

$E_2 \equiv x^2 + y^2 + z^2 - 10x + 15y - 25z = 0$

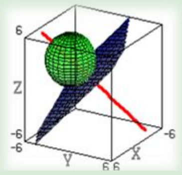
Prove that the two spheres are secant. Determine the plane that contains the intersection of the two spheres. Determine the centre and radius of the intersecting circle.



27 Prove that the point T (1,0,1) belongs to the plane:

$\pi \equiv x - 2y + 2z = 3$

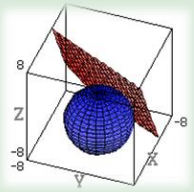
Determine the equation of the sphere that passes through the point P (1,0,5) and is tangent to T to the π plane.



28 Determine the equation of the plane tangent to the sphere:

$(x-3)^2 + (y-1)^2 + (z+2)^2 = 24$

passing through point M (-1,3,0)



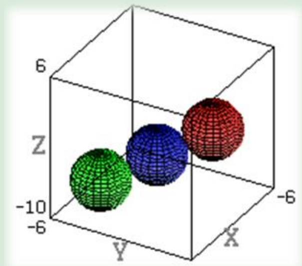
29

30

Let the sphere:

$x^2 + y^2 + z^2 - 6x - 4y + 8z + 20 = 0$

Find the sphere of equal radius, exterior tangent at point A (1,4, -3) of the sphere. Find the sphere of equal radius, exterior tangent at the point diametrically opposite to point A on the sphere.



MAY

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.

2

3

Sisters Laia and Aitana write their ages, one after the other, and obtain a four-digit number that is the square of their father's age. Nine years later they rewrite their ages in the same way and it happens again that it is the square of their father's age. What is the age of Laia, Aitana and her father?

4 We have two hourglasses, one of 7 minutes and the other of 11 minutes. What is the fastest method to control the cooking of a stew that should last 15 minutes?

5

6

The town of Benirredrà has a very strange set of speed limits. One kilometre from the centre of town there is a sign that reads 120 km / h, ½ kilometre from the centre, a sign says 60 km / h, 1/3 of a km from the centre there is a sign that says 40 km / h, at ¼ km from the centre there is a sign that says 30 km / h, at 1/5 from the centre a warning of 24 km / h and at a distance of 1/6 of km the warning says 20 km / h. If you always travel at the speed limit, how long will it take to get to the centre of town from the first advertisement?

7 Find five consecutive integers such that the sum of squares of the first three equals the sum of squares of the last two

1/8

9 A month ago 10% of a population had a disease and 90% did not. After the month, 10% of the sick people are cured and 10% of the people who did not have it became sick. What% of the population does not have the disease?

10

11

A horse and a mule walked together, carrying heavy sacks on their backs. The horse lamented his annoying burden, to which the mule said: What are you complaining about? If I took a sack from you, my burden would be double yours. On the other hand, if I give you a sack, your load will be equal to mine. How many bags does each carry?

12 An NGO sells 140 tickets to raise funds. Some tickets were sold at the original price (a whole), but the others were sold at half the initial price. € 2001 was raised. What is the starting price of the tickets?

13

14

At each station of a railway network, as many different tickets are sold as there are stations to which you can go or from which you can come (the outbound and return tickets are different). A new line with several stations is inaugurated and that forces to print 34 new tickets. How many stations were there and how many have been inaugurated?

15

16

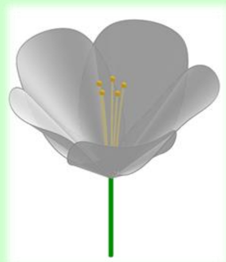
17

Two players, A and B, take turns playing the following game: they have a pile of 2021 stones. On his first turn, A chooses a divisor of 2021 and removes that number of stones from the pile. Next, B chooses a divisor of the number of stones remaining and removes that number of stones from the pile, and so on. The player who removes the last stone loses. Show that one of the players has a winning strategy and describe that strategy

18

In cuboland, the planets are cubes. Aitana has a 1 km edge. If its atmosphere is 500 m high at all points, what is the volume of the atmosphere?

19



20

The selling price of a coat was 40% less than the manufacturer's suggested price. Laia bought the coat for half the sale price. By what percentage is the value Laia paid for the coat less than the manufacturer's suggested price?

21

22

23 Find, if possible, the natural greatest and least whose sum of digits is 2022

24

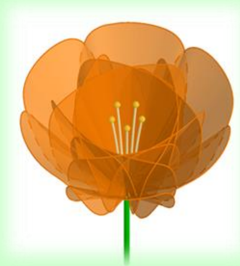
25

The rectangle in the figure is divided into nine squares. Calculate its height and its length knowing that the smallest square has a side 2 cm

26

A person has € 500 in a checking account at a bank. He can make two movements indefinitely, as long as he has money in the account: withdraw € 300 or deposit € 198. What is the maximum amount of money you can withdraw from account?

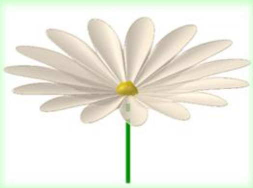
27



28 In the vertices of a quadrilateral a secret number is written in invisible ink and in visible ink the sum of the invisible numbers of the other three vertices. Can you give a rule to calculate the invisible numbers from the visible numbers?

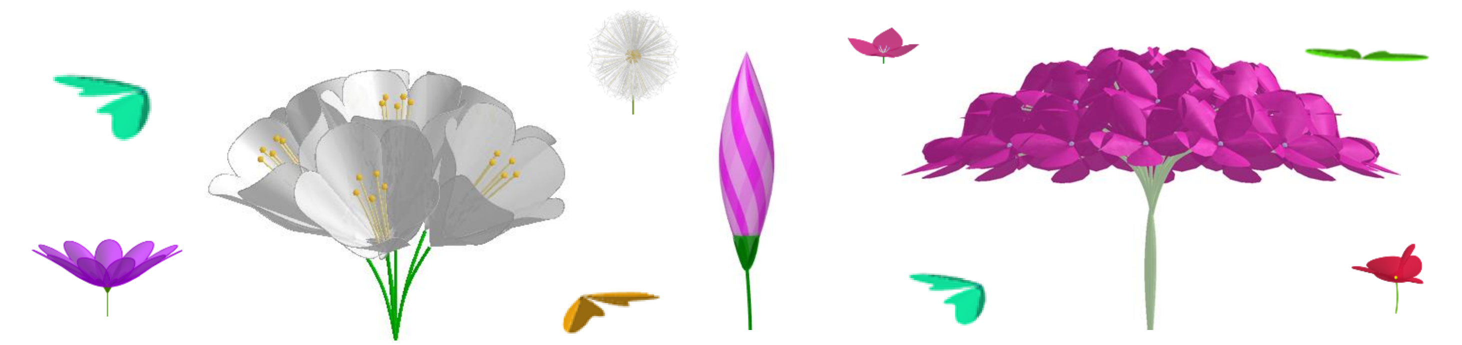
29

30



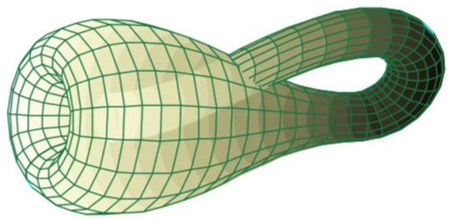
31

A basketball club has a men's section and a women's section. The arithmetic mean of the weight of the boys in the men's section is 90 kilos, the arithmetic mean of the weight of the girls in the women's section is 65 kilos. The arithmetic mean of the weight of all the members of the club is 75 kilos. Are there more girls than boys? What proportion of girls are there among all the players in the club?

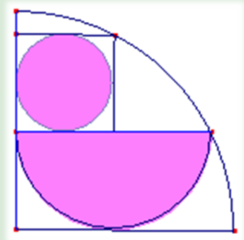


JUNE

MONDAY | TUESDAY | WEDNESDAY | THURSDAY | FRIDAY | SATURDAY | SUN.



1



Calculate the ratio of the shaded area to the area of the quadrant

2

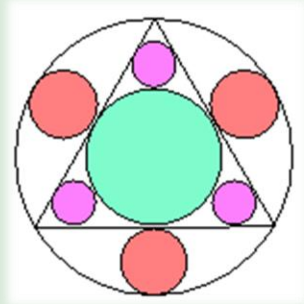
An equilateral triangle has been inscribed on a circle of radius R.

Seven circles have been drawn.

Find the radius of the circles.

Sangaku. Chiba prefecture

3



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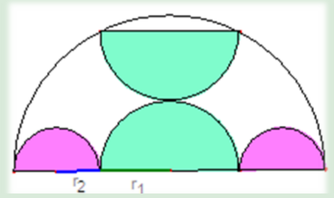
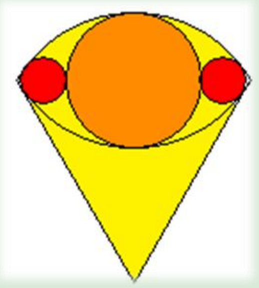


In the figure, calculate: $\frac{r_1}{r_2}$

5

6



7

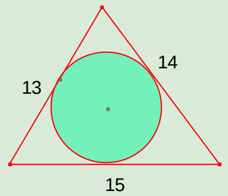
In the figure, the radius of the upper arc is the diameter of the orange circle.

Determine the ratio between the radii of the orange and red circles.

Sangaku. Tochigi Prefecture

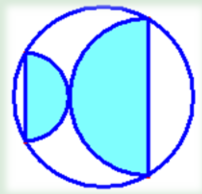
8 The sides of a triangle are 13,14,15.

Find the radius of the inscribed circle.



9 The diameters of the semicircles are parallel.

Find the ratio of the shaded area to the area of the circle.

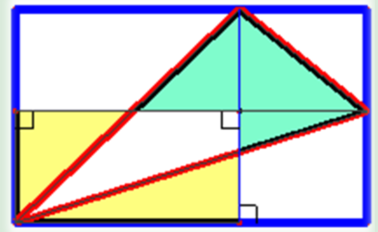


10

The area of the red triangle is one third of the outer blue rectangle.

Find the ratio of the area painted green to that painted yellow.

11



12

13 The two circles have radius 4.

Calculate the radius of the semicircle

14

15

In the figure, the radius of the semicircle is R = 1.

Find the radius of the four types of circumference.

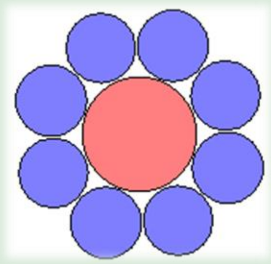
Fukushima prefecture

16

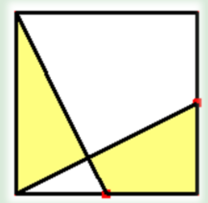
Eight circles are exterior tangents two to two and all are exterior tangents to one another.

Calculate the ratio between the radii of the two types of circles and the ratio between the areas of the sum of the blue eight and the red one

17



18 The points marked are the midpoints of the sides of the square. Find the ratio of the areas of the shaded region and the square



19

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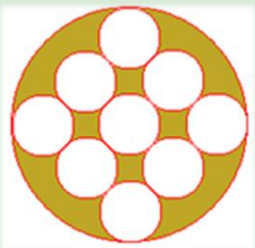


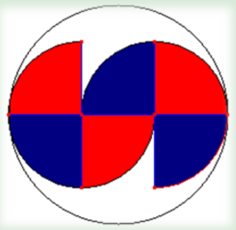
21

Nine equal circles tangent two by two are inside another circle.

Find the ratio between the areas of the sum of the nine circumferences and the outer circumference.

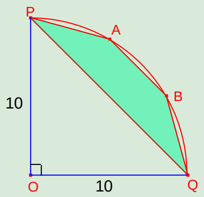
Shisouka Prefecture

22



Calculate the ratio between the area of the shaded area and the area of the outer circle

23



Let the quadrant of radius 10. Let A, B be the points of the arc such that PA = AB = QB. Find the area of the quadrilateral PABQ

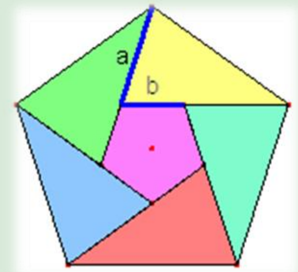
24

The regular pentagon in the figure has been divided into five triangles and one regular pentagon.

All six regions have the same area. Calculates

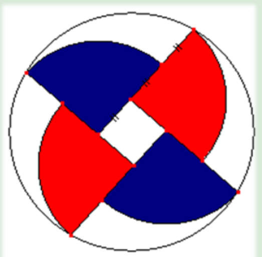
$\frac{a}{b}$

25

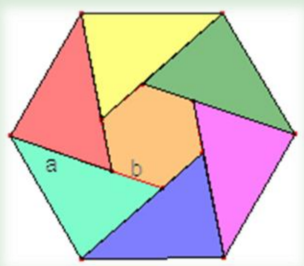


26

27 Calculate the ratio between the area of the shaded area and the area of the outer circle



28

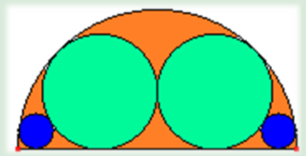


29

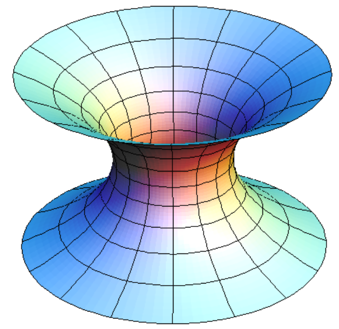
All seven regions in the figure have the same area. Calculate:

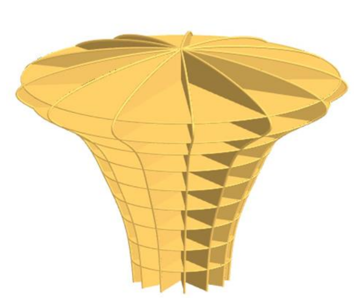
$\frac{a}{b}$

30



Given the semicircle of radius R, calculate the radii of the other circles.

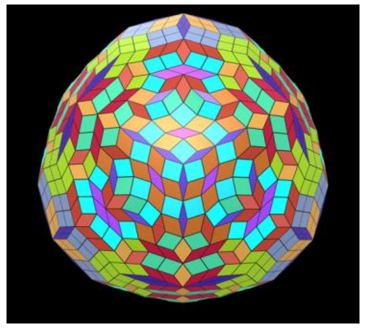


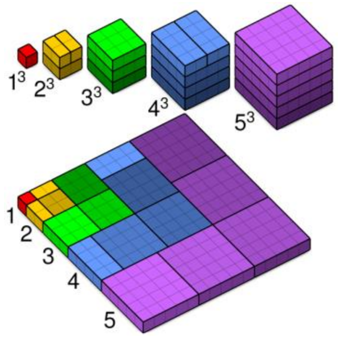


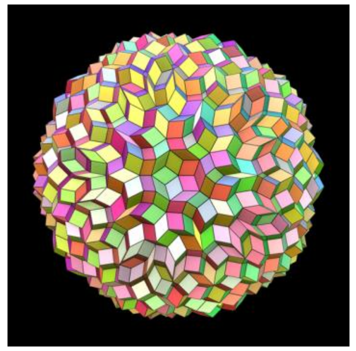
AUTHOR: RICARD PEIRÓ I ESTRUCH. IES “Abastos”. València

JULY

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	SUN.







1

Through the intersection points of the line

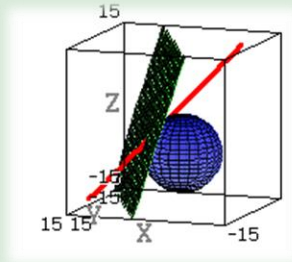
$$r \equiv \begin{cases} x = -5 + 3t \\ y = -11 + 5t \\ z = 9 - 4t \end{cases}$$

and from the sphere of equation

$$E \equiv (x+2)^2 + (y-1)^2 + (z+5)^2 = 49$$

planes tangent to the sphere have been drawn. Determine their equations.

2



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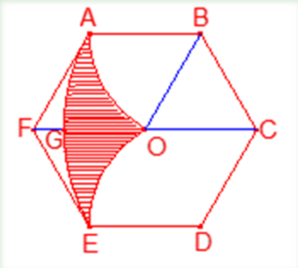
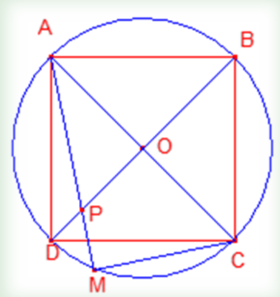
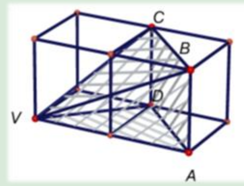


5 Let ABCDEF be a regular hexagon with centre O and side c. From B and D and with radius c two arcs are drawn: AO and EO. With centre at C and radius AC, the arc AGE is drawn. Find the area of the shaded area

6

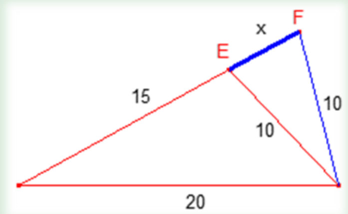


7



Let be two equal cubes joined by one face. Determine the ratio between the volume of the pyramid ABCDV and the sum of the volumes of the two cubes

8 In the figure, calculate the measure of segment EF



9

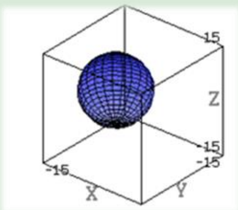
In a regular hexagon ABCDEF another regular hexagon GHIJKL is inscribed such that:

$$\overline{AG} = \frac{1}{3}\overline{AB}$$

Calculate the ratio between the areas of the hexagons

10

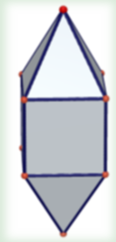
11



Determine the equation of the sphere that passes through the points A (1, -2, -1); B (-5.10, -1); C (4,1,11) and D (-8, -2,2)

12

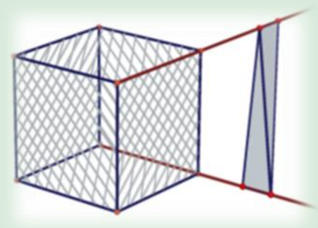
The figure is made up of a cube with edge a and two pyramids with a square base and height a. Determine area and volume of the body



13

Square ABCD is inscribed in a circle of radius 30. Chord AM measures 50 and intersects diagonal BD at point P. Find the measure of segment AP

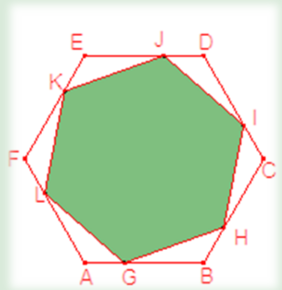
14



15

Two intersecting edges of a cube are extended. In each extension segments of length 1 are taken. Where must these segments be located so that the volume of the tetrahedron formed by the four ends of the segments is maximum?

16



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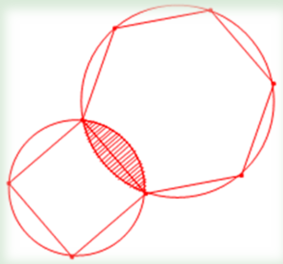


19

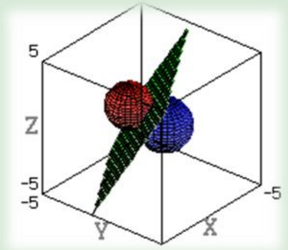
Given the spheres:

$$E_1 \equiv 2x^2+2y^2+2z^2+3x-2y+z-5=0$$

$$E_2 \equiv x^2+y^2+z^2-x+3y-2z+1=0$$

determines the relative position of E_1 and E_2. If they are secants, find the plane where they intersect. Determine the centre and radius intersection of the spheres

20



21

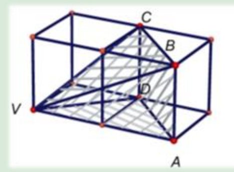
Let the sphere be given:

$$x^2+y^2+z^2+6y-4z+9=0$$

Calculate the equation of the sphere concentric with it that is tangent to the plane:

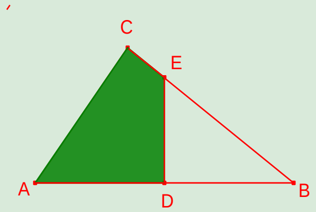
$$2x-3y+2z+4=0$$

22



Let be two equal cubes joined by one face. Find the total area of the pyramid ABCDV

23 ΔABC is a right triangle in C. D is the midpoint of AB and DE⊥AB. If AC=12 and AB=20calculate the area of ADEC

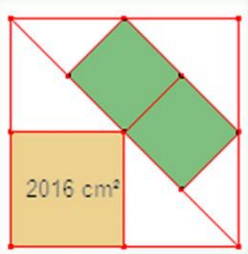


24

25

On one side of a regular hexagon with side c a square has been drawn. Find the area of the intersection of the two circles circumscribed to the regular polygons

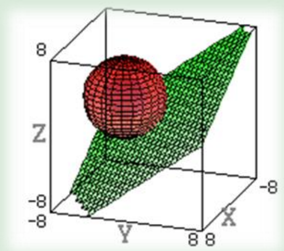
26



27

A square has been divided into two triangles on the diagonal. A square with area 2016 cm^2 has been inscribed in the lower triangle and two equal squares have been inscribed in the upper triangle. Find the area of one of those squares

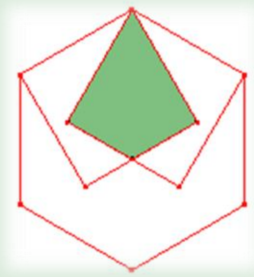
28



29

30

On two consecutive sides of a regular hexagon, two squares have been drawn towards the inside. Determine the ratio between the area of the common area of the two squares and the area of the initial hexagon



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