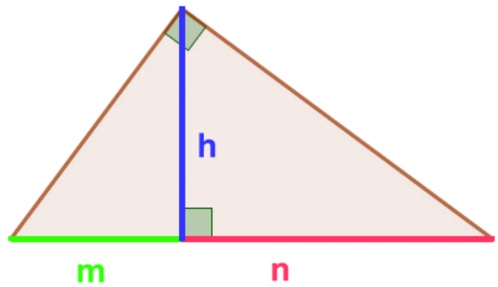
SOLUTIONS SEPTEMBER 2021

TWITTER GEOMETRY PROBLEMS (3rd and 4th year of E.S.O. and high school). MIGUEL HERRAIZ HIDALGO SOLUTIONS. SES of Cabanes. Castelló**.**

Small compendium of geometric results used.

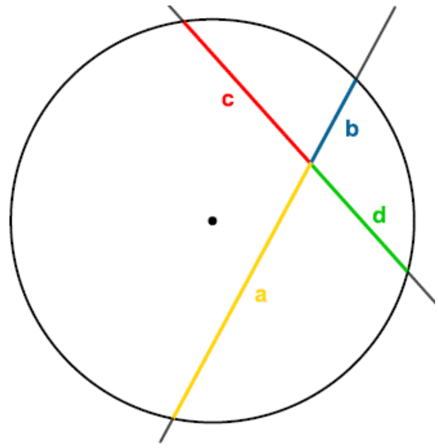
Height theorem:

$$h^2 = m \cdot n$$



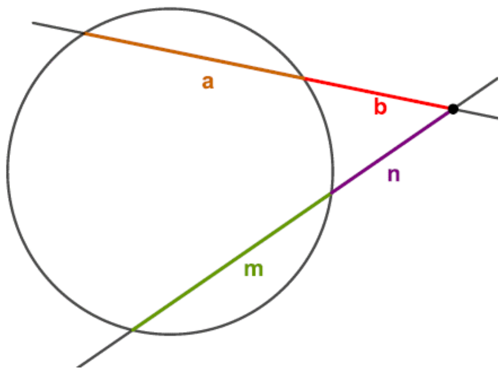
String theorem (problem of day 29):

$$a \cdot b = c \cdot d$$



Power of a point with respect to a circumference:

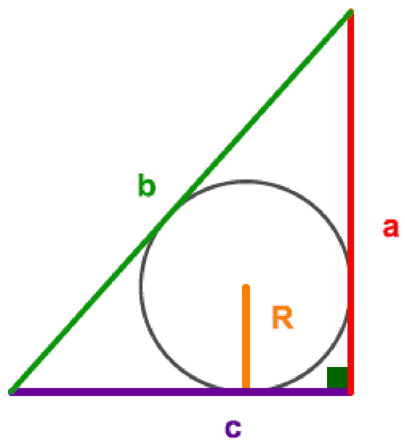
$$(a + b) \cdot b = (m + n) \cdot n$$



Relationship between area and perimeter of a right triangle and the radius of the circle inscribed in the triangle (problem of day 13):

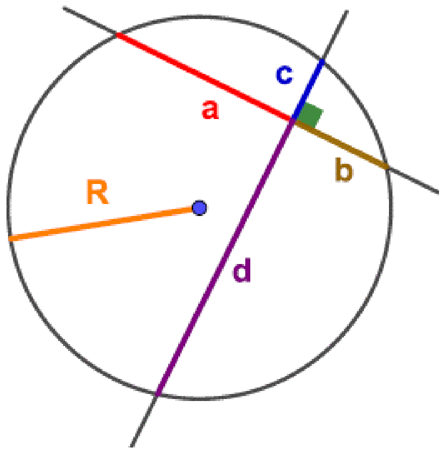
$$c \cdot a = (a + b + c) \cdot R$$

$$2A = P \cdot R$$



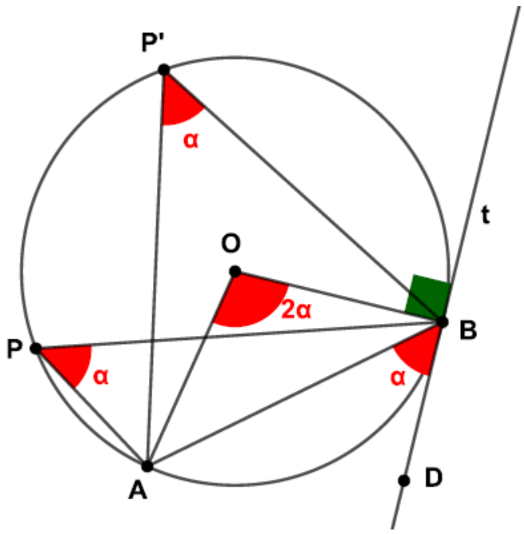
Trigonometric ratios of the double angle:

$$\cos 2\alpha = \cos^2\alpha - \mathrm{sen}^2\alpha; \quad \mathrm{sen}2\alpha = 2\mathrm{sen}\alpha\cos\alpha; \quad \mathrm{tg}2\alpha = \frac{2\mathrm{tg}\alpha}{1 - \mathrm{tg}^2\alpha}$$



Faure's theorem (problem of day 27):

$$4R^2 = a^2 + b^2 + c^2 + d^2$$



Central angle and inscribed angle (t is the tangent to the circle at B)

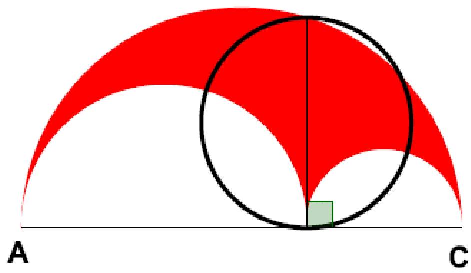
$$2\alpha = \widehat{AB}$$

Inscribed angle:

$$\angle APB = \angle AP'B = \angle ABD = \alpha$$

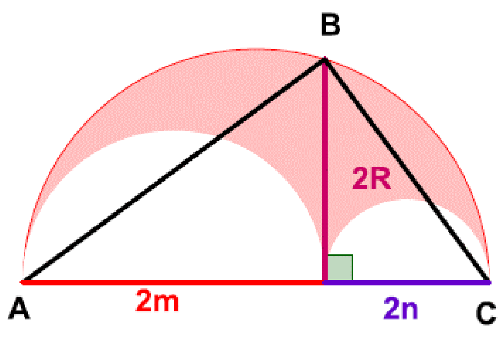
Centre angle:

$$\angle AOB = 2\alpha$$



<u>September 1:</u> Three semicircles cantered on segment AC. Compare the area of the red zone and the area of the black circle.

<u>Solution:</u>



Consider the triangle ΔABC. That triangle is right at B (since AC is a diameter of the large semicircle), with hypotenuse 2n + 2m (m and n being the radii of the medium and small semicircles) and with altitude from B of 2R (R being the radius of the black circle).

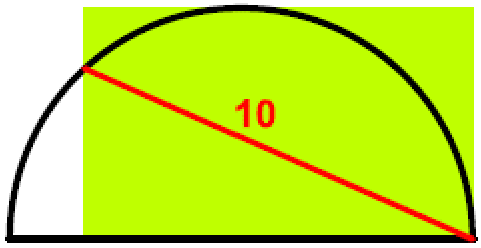
If we apply the altitude theorem to the triangle ΔABC, we have:

$$2m \cdot 2n = (2R)^2 \quad \Rightarrow \quad m \cdot n = R^2$$

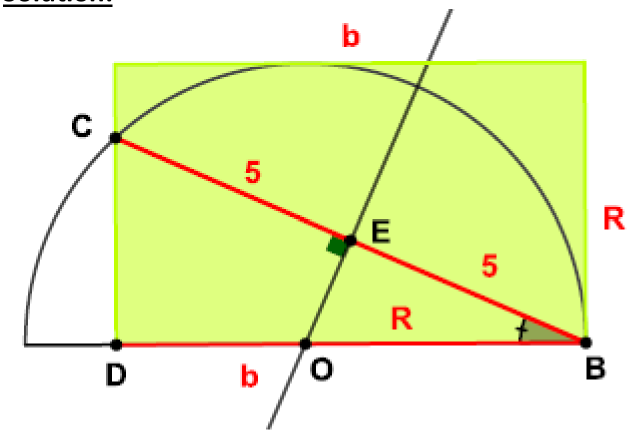
Therefore:

$$\textbf{red zone area} = \frac{\pi[(m+n)^2 - m^2 - n^2]}{2} = \frac{2\pi mn}{2} = \pi R^2 = \textbf{black circle area}$$

September 2: Semicircle, string of length 10, and rectangle. Find area of the rectangle.



Solution:

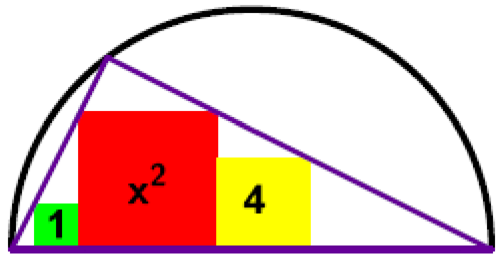


Let b be the base of the rectangle and R its height.

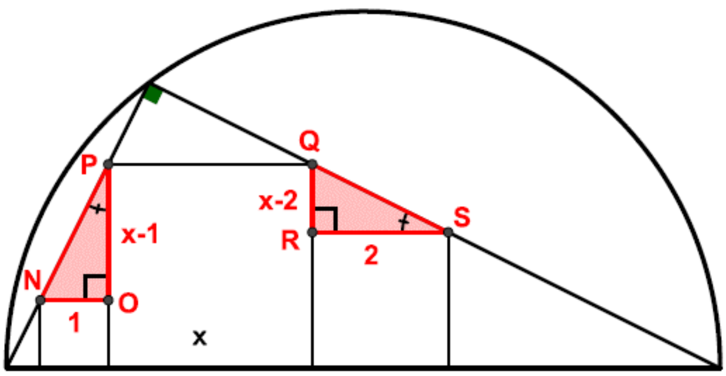
We draw through the midpoint of the chord (point E) a perpendicular to the chord. That perpendicular passes through the center of the semicircle. The triangles ΔEOB and ΔDBC are generated. Both are rectangles (at E and at D) and have the angle at B in common. Therefore they are similar. From here:

$$\frac{R}{5} = \frac{5+5}{b} \quad \Rightarrow \quad Rb = 50$$

September 3: Semicircle, three squares with areas 1, x^2 and 4 and a triangle. Find x



Solution:

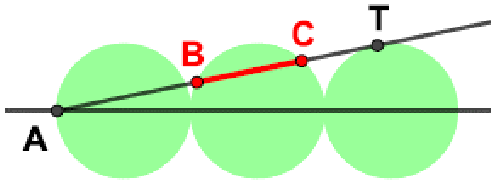


The triangles ΔOPN and ΔRSQ are similar since they are right-angled (at O and at R) and the indicated angles are equal (because they have perpendicular sides). Therefore:

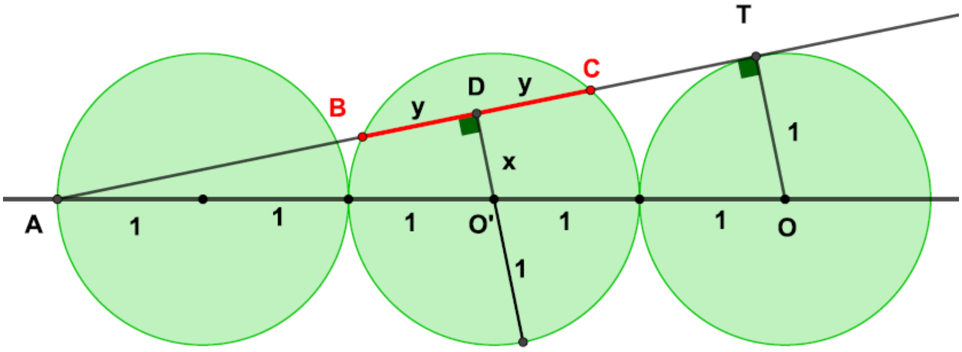
$$\frac{1}{x-1} = \frac{x-2}{2} \quad \Rightarrow \quad x(x-3) = 0$$

Therefore x = 3, (because x = 0 makes no sense)

September 4: Three equal circles, radius 1, tangent to each other, T point of tangency. Find BC



Solution:



Consider D, the midpoint of segment BC. The perpendicular through D to segment BC passes through the center of the intermediate circle: O'.

We have to ΔATO ≈ ΔADO' since both are rectangles (at T and D) and have the angle at A in common. Therefore:

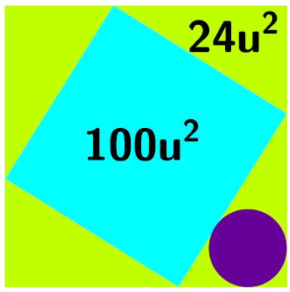
$$\frac{5}{1} = \frac{3}{x} \Rightarrow x = \frac{3}{5}$$

By the theorem of the chords in the intermediate circle, we will have:

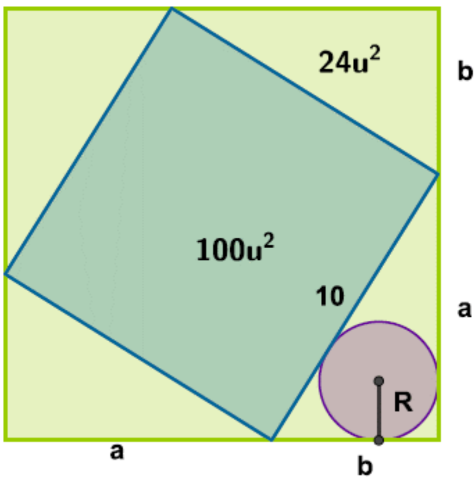
$$y \cdot y = \left(1 - \frac{3}{5}\right) \cdot \left(1 + \frac{3}{5}\right) \Rightarrow y^2 = \frac{16}{25} \Rightarrow y = \frac{4}{5}$$

$$\mathbf{BC} = 2y = \mathbf{\frac{8}{5}}$$

September 6: Two squares and a circle. Find the area of the circle



Solution:



The four green triangles are the same (having the legs measures a and b and hypotenuse 10). The area of the big square is:

$$100 + 4 \cdot 24 = 196 = 14^2 \Rightarrow a + b = 14$$

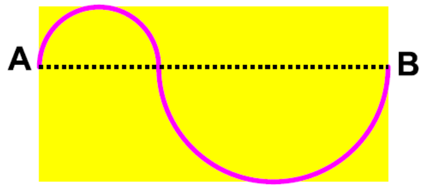
By the relationship between the area and perimeter (A and P) of a right triangle and the radius of the inscribed circle (R) (problem of day 13), we have:

$$2A = P \cdot R \Rightarrow 2 \cdot 24 = (10 + a + b) \cdot R \Rightarrow R = \frac{2 \cdot 24}{(10 + 14)} = 2$$

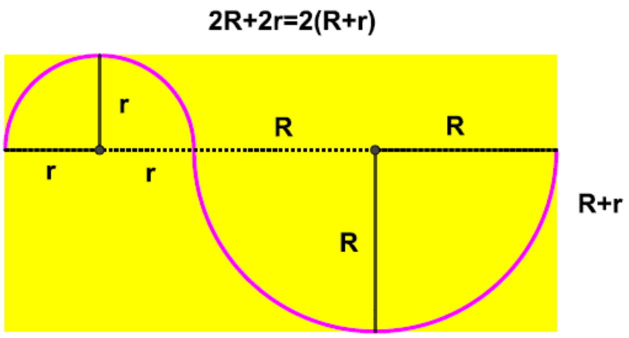
The area of the circle is then:

$$\textbf{circle area} = \pi \cdot 2^2 = \mathbf{4\pi}$$

September 7: Two semicircles with diameters in AB||base. If the fuchsia line measures π, find the perimeter of the rectangle



Solution:



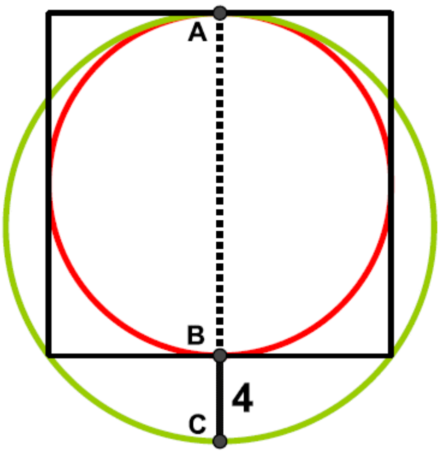
The length of the two semicircles is:

$$\frac{2\pi r}{2}+\frac{2\pi R}{2}=\pi \Rightarrow \quad R+r=1$$

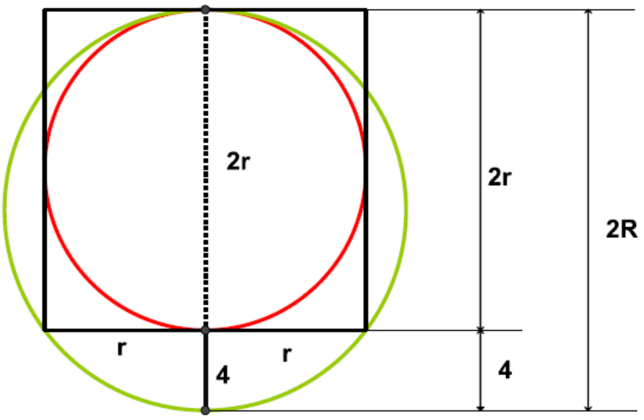
For the yellow rectangle, we will have:

$$\left.\begin{array}{r}\text{Altura } = R+r=1 \\ \text{base}=2(R+r)=2\end{array}\right\} \Rightarrow P=2\cdot(1+2)=6$$

September 8: Two circles with center at AB. If BC = 4 cm, find the area enclosed between the circles.



Solution:



If we apply the chord theorem to the two chords of the green circle we have:

$$2r\cdot 4=r\cdot r \quad \Rightarrow \quad r=8$$

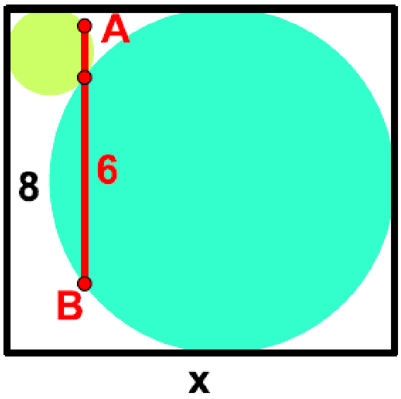
From the figure:

$$2r+4=2R \quad \Rightarrow \quad R=r+2=8+2=10$$

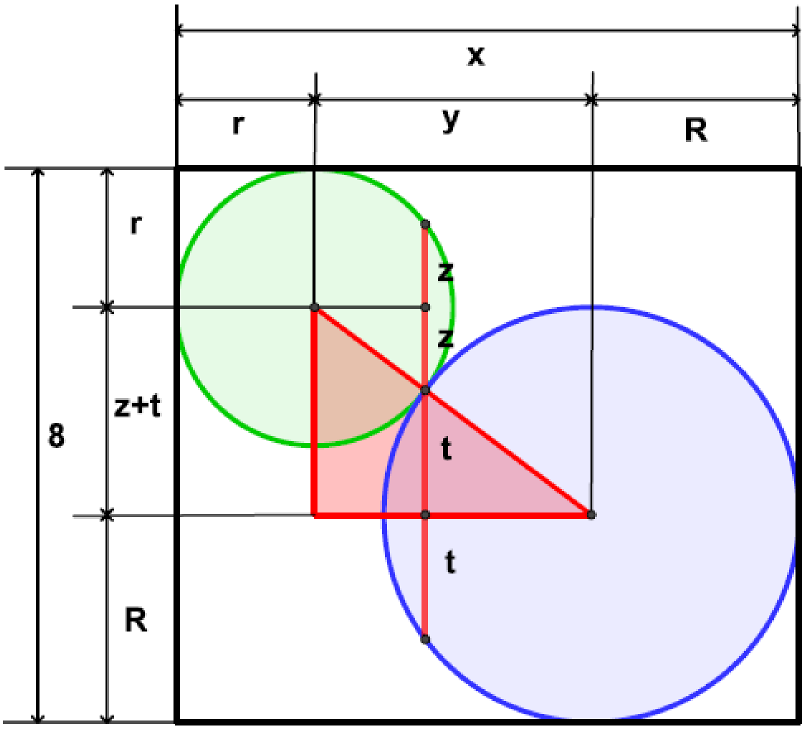
Therefore, the area between the two circles is:

$$\pi R^2-\pi r^2=\pi(10^2-8^2)=36\pi$$

September 9: Two tangent circles and a rectangle with base x and height 8. If AB = 6, find x.



Solution:



From the figure we have:

$$x = y + r + R$$

Since the segment AB measures 6:

$$2t + 2z = 6 \Rightarrow t + z = 3$$

Plus:

$$r + t + z + R = 8 \Rightarrow r + R = 8 - 3 = 5$$

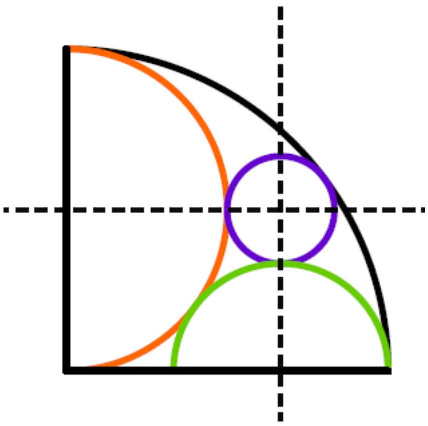
Consider the red triangle, which is a right triangle with legs: t + z = 3 and y, and hypotenuse r + R = 5. Applying the Pythagorean theorem to it, we have:

$$y = \sqrt{5^2 - 3^2} = 4$$

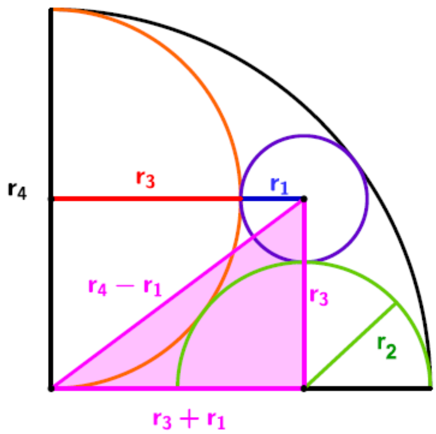
So:

$$x = y + r + R = 4 + 5 = 9$$

September 10: A quadrant, a circle and two semicircles, all tangent to each other. Find the relationship between radii



Solution:



Let: r_1 be the radius of the small circle, r_2 and r_3 the radii of the small and large semicircles and r_4 the radius of the quadrant. Obviously:

$$2r_3 = r_4 \;\; ; \;\; r_3 = r_1 + r_2 \; (*) \;\; ; \;\; r_3 + r_1 = r_4 - r_2$$

Consider the magenta right triangle: Applying the Pythagorean theorem we have:

$$(r_4 - r_1)^2 = (2r_3 - r_1)^2 = r_3^2 + (r_1 + r_3)^2 \quad \Rightarrow \quad 2r_3^2 = 6r_3r_1$$

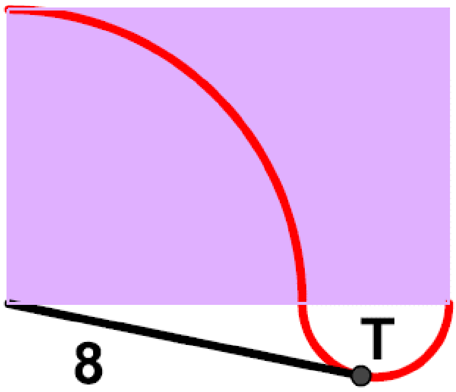
Neglecting $r_3 = 0$, we are left with $r_3 = 3r_1$. Substituting in (*), we have:

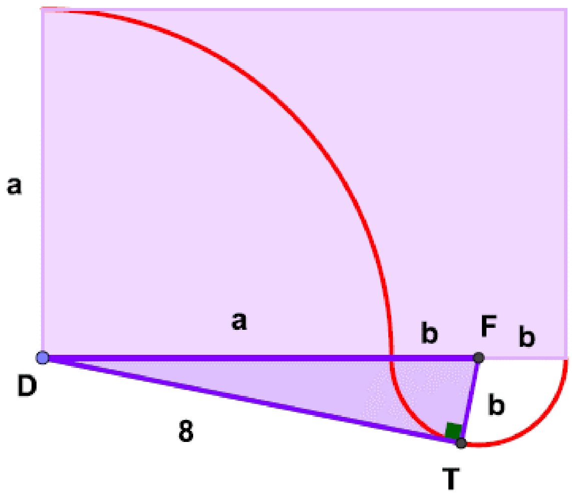
$$r_3 = r_1 + r_2 = \frac{r_3}{3} + r_2 \quad \Rightarrow \quad \mathbf{3r_2 = 2r_3}$$

With that:

$$r_4 = 2r_3 = 3r_2 = 6r_1 \quad \Rightarrow \quad \mathbf{r_4 : r_3 : r_2 : r_1 = 6 : 3 : 2 : 1}$$

September 11: A quadrant, a semicircle, a rectangle, T is a point of tangency. Find the area of the rectangle





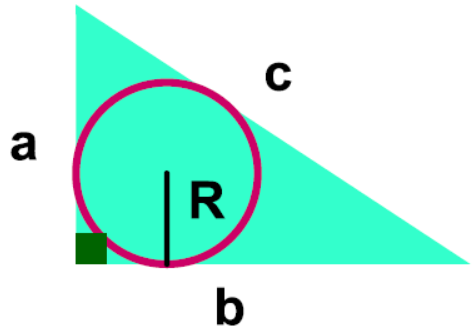
Solution 1: Let us consider the right triangle ΔDTF. Applying Pythagoras, we will have:

$$(a + b)^2 = b^2 + 64 \quad \Rightarrow \quad a^2 + 2ab = 64$$
$$\Rightarrow \quad \text{altura} \cdot \text{base} = A_{rec} = 64$$

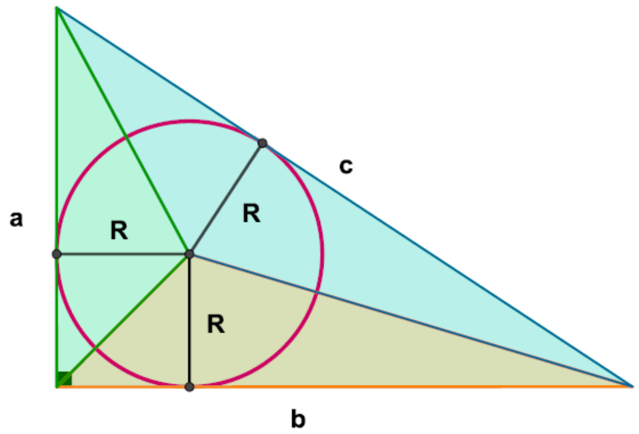
Solution 2: If we calculate the power of point D with respect to the semicircle, we have:

$$8^2 = a \cdot (a + 2b) = A_{rec}$$

September 13: (Relationship between area and perimeter of a right triangle and radius of the inscribed circle) Find R as a function of a, b and c.



Solution:



The center of the circumscribed circle (of radius R) divides the initial triangle into three triangles with base the sides of the initial triangle and height R. Then:

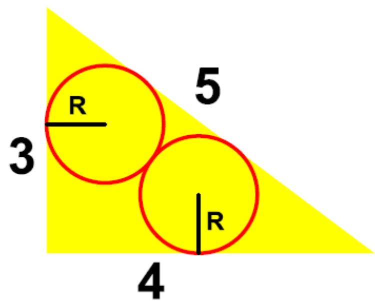
$$\frac{ab}{2} = A = \frac{aR}{2} + \frac{bR}{2} + \frac{cR}{2} = \frac{R(a+b+c))}{2}$$

In other words:

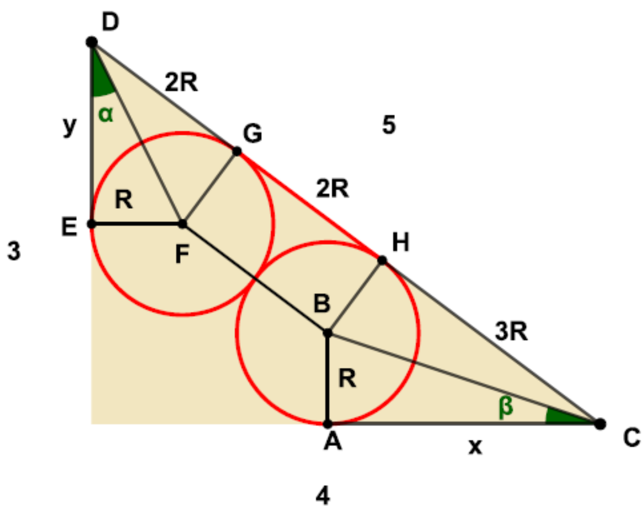
$$2A = R \cdot P$$

where A is the area and P the perimeter of the initial triangle and R is the radius of the inscribed circle

September 14: Find R



Solution 1:



Let F and B be the centers of the circles. Then, since F (B) is equidistant from the lines that generate the angle in D (C), we will have that F (B) is the bisector of the angle in D (C).

From the triangles ΔDEF and ΔABC we will have:

$$\text{tg}2\alpha = \frac{2\text{tg}\alpha}{1-\text{tg}^2\alpha} = \frac{4}{3} \Rightarrow 2\text{tg}^2\alpha + 3\text{tg}\alpha - 2 = 0 \Rightarrow \text{tg}\alpha = \frac{1}{2}$$

(despising tg $\alpha = -2$)

$$tg2\beta = \frac{2tg\beta}{1 - tg^2\beta} = \frac{3}{4} \Rightarrow 3tg^2\beta + 8tg\alpha - 3 = 0 \Rightarrow tg\beta = \frac{1}{3}$$

(despising tg β = – 3).

We have in ΔDEF:

$$\frac{1}{2} = tg\alpha = \frac{R}{y} \quad \Rightarrow \quad y = 2R$$

And in ΔABC:

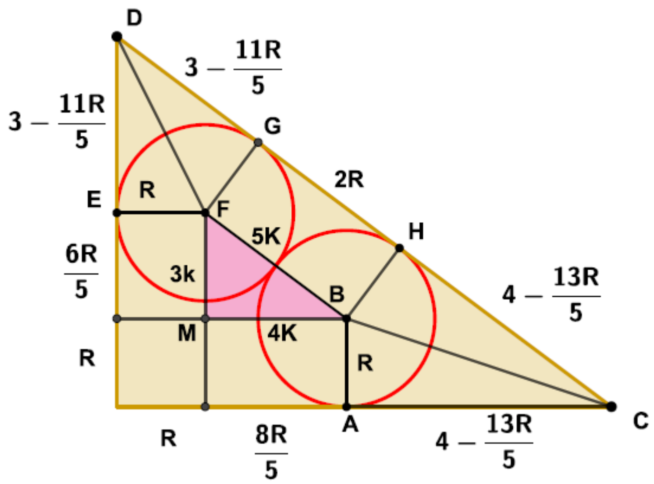
$$\frac{1}{3} = tg\beta = \frac{R}{x} \quad \Rightarrow \quad x = 3R$$

Furthermore: ΔDEF = ΔDFG because they are rectangles and have the same small legs and the same hypotenuse. (ΔABC = ΔHBC, for the same reason). Then: DG = 2R and HC = 3R.

Finally:

$$5 = 2R + 2R + 3R = 7R \quad \Rightarrow R = \frac{5}{7}$$

Solution (Toni Gomà):



Let F and B be the centers of the circles. The initial triangles and ΔMFB are similar since both have parallel sides. Therefore, FB = 5k =2R, MB = 4k and MF = 3k. So that:

$$5k = 2R \Rightarrow k = \frac{2R}{5} \Rightarrow \begin{cases} MB = \frac{8R}{5} \\ MF = \frac{6R}{5} \end{cases}$$

From here:

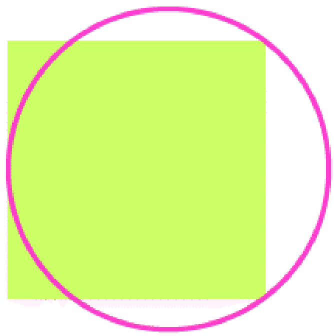
$$AC = 4 - \left(R + \frac{8R}{5}\right) = 4 - \frac{13R}{5}$$

$$ED = 3 - \left(R + \frac{6R}{5}\right) = 3 - \frac{11R}{5}$$

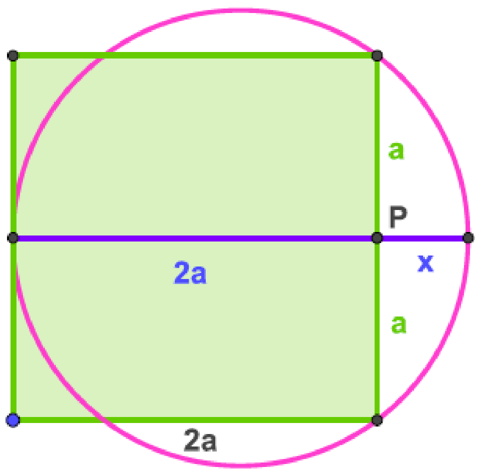
Furthermore: ΔDEF = ΔDFG because they are rectangles and have the same small legs and the same hypotenuse. (ΔABC = ΔHBC, for the same reason). Then, taking into account the hypotenuse of the initial triangle, we have:

$$5 = DG + GH + HC = 3 - \frac{11R}{5} + 2R + 4 - \frac{13R}{5} \quad \Rightarrow \quad R = \frac{5}{7}$$

September 15: A square and a circle. Which of the two has the larger perimeter?



Solution:



Let 2a be the side of the square. When calculating the power of the point P with respect to the circumference, we have:

$$a \cdot a = a^2 = 2a \cdot x \ \Rightarrow \ x = \frac{a}{2}$$

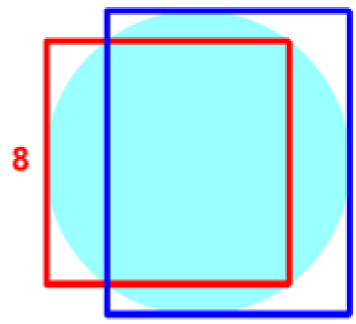
(since $a \neq 0$). Therefore, if R is the radius of the circle:

$$2R = 2a + x = 2a + \frac{a}{2} = \frac{5a}{2} \ \Rightarrow \ R = \frac{5a}{4}$$

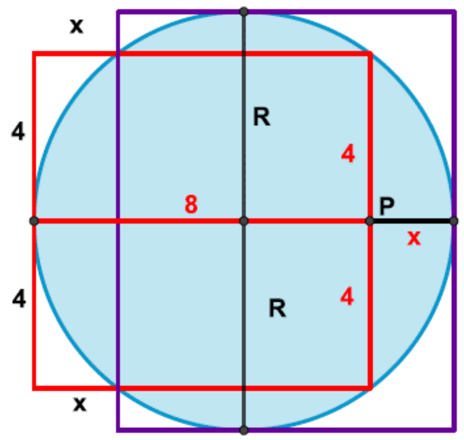
Finally:

$$P_{square} = 4 \cdot 2a = 8a > \frac{5a\pi}{2} = 2R\pi = P_{circle}$$

September 16: A square of side 8, a rectangle and a circle. Find the area of the circle and the rectangle.



Solution:



Calculating the power of P with respect to the circumference, we have:

$$4 \cdot 4 = x \cdot 8 \ \Rightarrow \ x = 2$$

Therefore the diameter of the circle is:

$$8 + 2 = 10 = 2R \ \Rightarrow \ R = 5$$

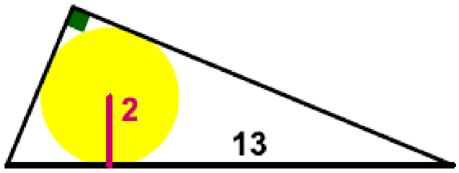
The area of the circle is:

$$\pi R^2 = 25\pi$$

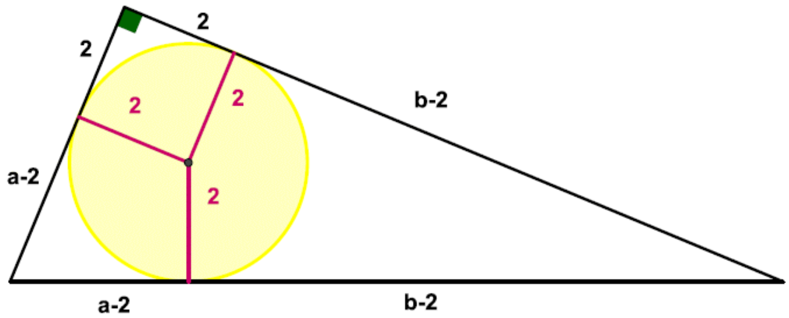
Finally for the rectangle, we have:

$$\left.\begin{array}{l} base = 8 - x + x = 8 \\ height = 2R = 10 \end{array}\right\} \Rightarrow Area = 8 \cdot 10 = 80$$

September 17: A right triangle with hypotenuse 13. Inscribed circle of radius 2. Find the area of the triangle



Solution:



Remember that the two tangents to the circumference through a point outside it measure the same. We will then have that if a (b) is the small (large) leg of the triangle, since the hypotenuse measures 13:

$$a-2+b-2=13 \Rightarrow a+b=17$$

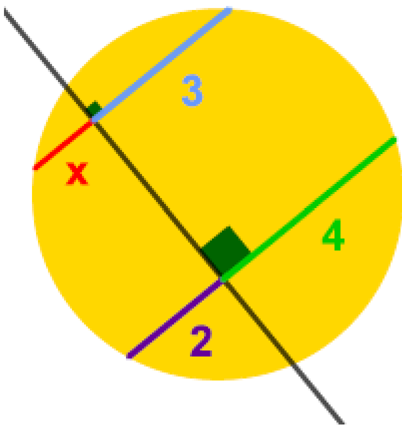
Since the Pythagorean theorem holds in the triangle, we will have the system:

$$\left.\begin{matrix} a+b=17 \\ a^2+b^2=13^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} (a+b)^2=17^2 \\ a^2+b^2=13^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a^2+b^2+2ab=17^2 \\ a^2+b^2=13^2 \end{matrix}\right\} \Rightarrow ab=\frac{17^2-13^2}{2}=60$$

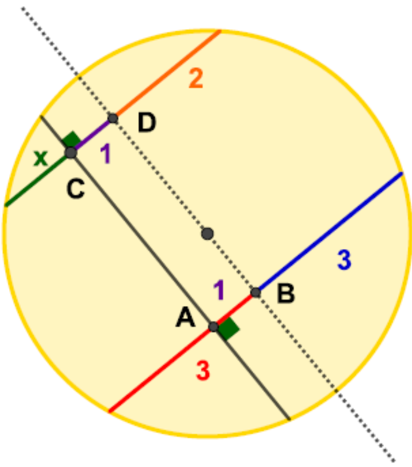
Then the area of the triangle is:

$$A=\frac{ab}{2}=\frac{60}{2}=30$$

September 18: Circle and three strings, two of them parallel. Find x



Solution:

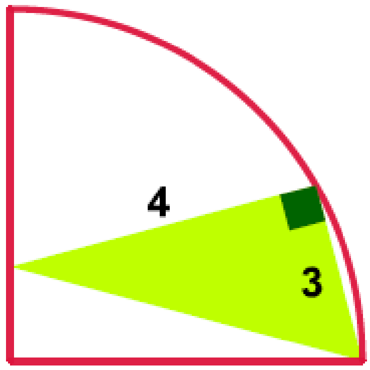


Remember that the diameter is the intersection with the circle of the perpendicular to a chord through its midpoint. Let the line through B and D be that perpendicular. What:

$$\frac{4+2}{2}=\frac{6}{2}=3 \quad \Rightarrow \quad AB=1=CD$$

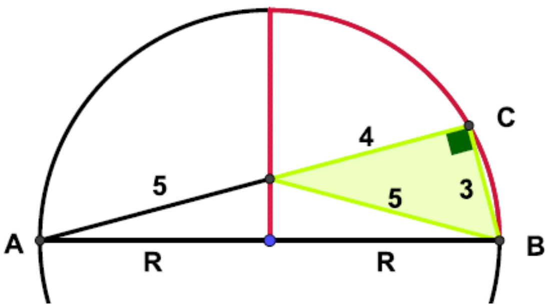
From where:

$$x+1=3-1=2 \quad \Rightarrow \quad x=1$$



September 20: A quadrant and a right triangle with legs 3 and 4. Find the area of the quadrant

Solution:



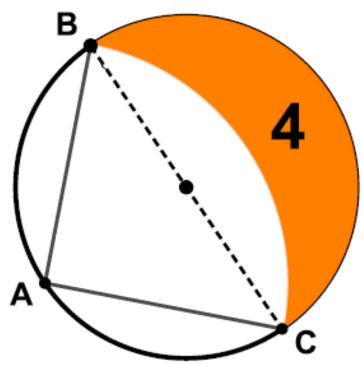
In the green triangle we will have that its hypotenuse is equal to 5.

Consider the circumference generated by the quadrant. Since the angle at C is 90°, AB will be the diameter of the circle. From here:

$$2R = \sqrt{(4+5)^2 + 3^2} = 3\sqrt{10} \Rightarrow R = \frac{3\sqrt{10}}{2}$$

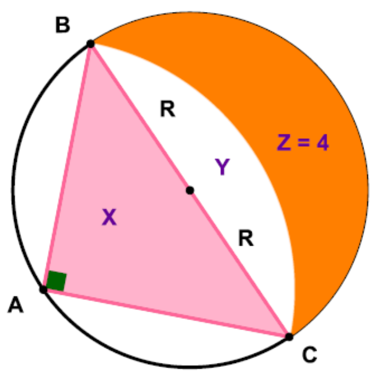
Therefore, the area of the quadrant will be:

$$\frac{\pi R^2}{4} = \frac{\pi \frac{90}{4}}{4} = \frac{45}{8}\pi$$



September 21: A circle of diameter BC and a quadrant. Area of the shaded area 4. Find the area of the circle

Solution:



Let R be the radius of the circle, X be the area of the triangle ΔABC, Y be the area of the segment that generates the quadrant, and Z = 4 be the area of the orange zone. In ΔABC, when applying Pythagoras (because AB = AC = radius of the quadrant), we will have:

$$AC = AB = \sqrt{2} \cdot R$$

So:

$$\left.\begin{aligned} X + Y &= \frac{1}{4}\pi\left(\sqrt{2}\,R\right)^2 = \frac{\pi R^2}{2} \\ Y + Z &= \frac{1}{2}\pi R^2 \end{aligned}\right\} \Rightarrow X + Y = Y + Z \Rightarrow X = Z = 4$$

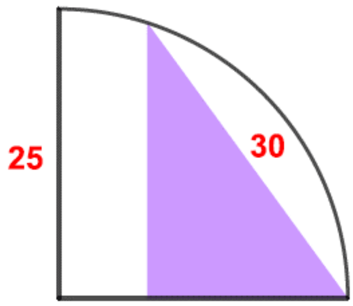
But:

$$4 = X = \frac{AB \cdot AC}{2} = R^2$$

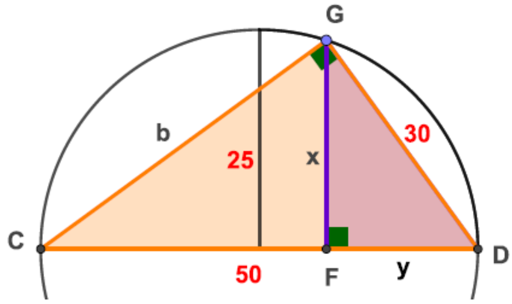
So R = 2. And from here:

$$A_{circle} = \pi \cdot 2^2 = 4\pi$$

September 22: Quadrant with radius 25 and right triangle with hypotenuse 30. Find the legs of the triangle



Solution:



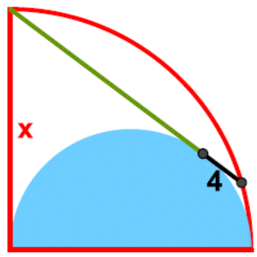
Consider the circumference generated by the quadrant. We draw the segment CG. Since CD is a diameter (of length 2·25 = 50), the angle at G is 90°. By applying Pythagoras in the ΔCDG, we will have:

$$b = \sqrt{50^2 - 30^2} = 40$$

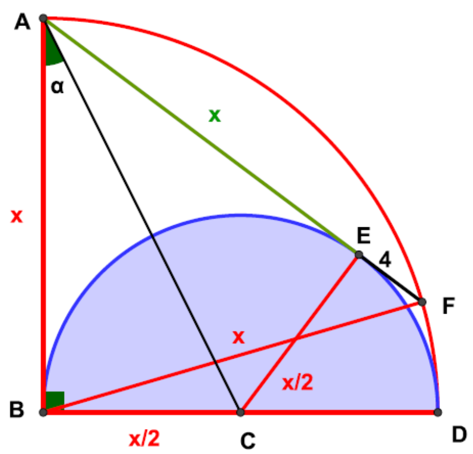
Since ΔCGD ≈ ΔGFD (as both are rectangles and have the angle in D in common), we will have:

$$\frac{x}{40} = \frac{30}{50} = \frac{y}{30} \Rightarrow \begin{cases} x = 24 \\ y = 18 \end{cases}$$

September 23: A quadrant, a semicircle, and a tangent chord. Find x



Solution 1:



Let us consider the triangle ΔABF, with sides x, x and x + 4. If we knew any angle of the triangle we could apply the law of cosines, to find x.

Let C be the center of the semicircle. Consider the two tangents to the semicircle drawn by A and the bisector of the angle formed at A. Since the bisector is equidistant from the lines that generate the sides of the angle, we will have that C belongs to the bisector and two equal triangles are generated: ΔABC and ΔAEC. In the triangle ΔABC, we will have:

$$AB = x; BC = \frac{x}{2}; \; AC = \frac{\sqrt{5}}{2}x$$

with what:

$$\cos\alpha = \frac{2}{\sqrt{5}};\ \ \mathrm{sen}\alpha = \frac{1}{\sqrt{5}};\ \ \cos 2\alpha = \cos^2\alpha - \mathrm{sen}^2\alpha = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

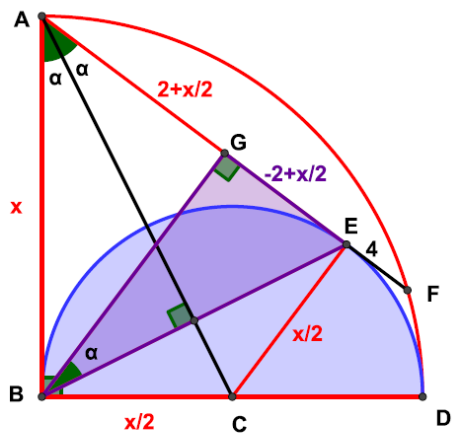
Applying the cosine rule on ΔABF, we have:

$$x^2 = x^2 + (x+4)^2 - 2x(x+4)\frac{3}{5}$$

which leads to:

$$\left.\begin{array}{c} x+4=0 \ \text{ that is despised} \\ x+4 = \dfrac{6x}{5} \Rightarrow x = 20 \end{array}\right\}$$

Solution 2 (Toni Gomà):



As the tangents to a circumference through a point outside it measure the same, we will have that ΔABE is isosceles since AB = AE = x. Also, the bisector by A is the height (and the median and perpendicular bisector) of ΔABE, so AC is perpendicular to BE.

Let us draw through B the perpendicular to AF, generating the point G. Since AF is a chord of the quadrant, G divides AF into two equal parts. Later:

$$AG = GF = \frac{x+4}{2} = \frac{x}{2} + 2 \ \Rightarrow \ GE = \frac{x}{2} - 2$$

Furthermore, α = ∠EBG since it has sides perpendicular to ∠CAG = α. So: ΔABC ≈ ΔBGE (because they are rectangles and both have an angle α)

Since in ΔABC one leg is twice the other we will have to:

$$BG = 2\left(\frac{x}{2} - 2\right) = x - 4$$

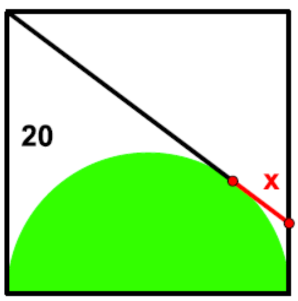
Finally, in the triangle ΔAGB the Pythagorean theorem is fulfilled (because it is a rectangle):

$$x^2 = \left(\frac{x}{2} + 2\right)^2 + (x-4)^2 \ \Rightarrow \ x^2 - 24x + 80 = 0 \ \Rightarrow \ x = \begin{cases} 4 \\ 20 \end{cases}$$

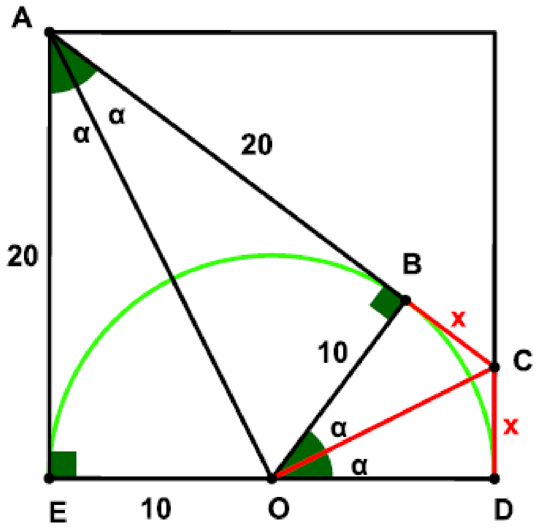
The solution x = 4 is neglected, since with it:

$$GE = \frac{x}{2} - 2 = 0$$

September 24: Square of side 20, semicircle and tangent chord. Find x



Solution:



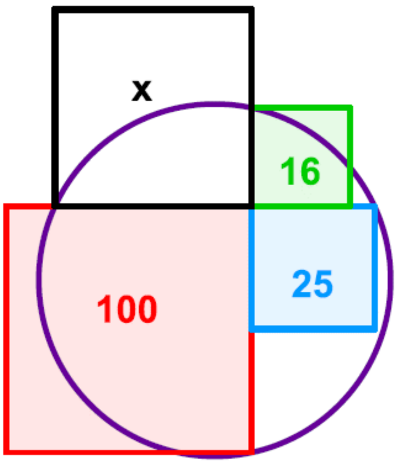
Remember that the tangents through an exterior point to a circumference measure the same. So:

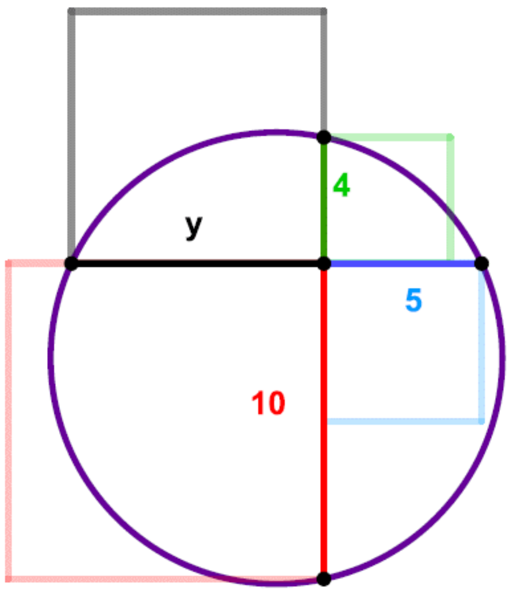
$$AE = 20 = AB; \quad BC = x = CD$$

On the other hand, the center of the equidistant semicircle of the tangents, then belongs to the bisector of the angle at A and at O. In addition, the angle at A and the angle at O measure the same because they have perpendicular sides. We will then have that $\Delta AEO \approx \Delta OBC$ (because both are rectangles and have an angle of α). So:

$$\frac{20}{10} = \frac{10}{x} \quad \Rightarrow \quad x = 5$$

September 25: Four squares of areas 100, 25, 16 and x. Find x and the area of the circle.





Solution 1: Applying the string theorem:

$$5y = 4 \cdot 10 \quad \Rightarrow \quad y = 8 \quad \Rightarrow \quad y^2 = x = 64$$

Applying Faure's theorem (since the chords are perpendiculars)

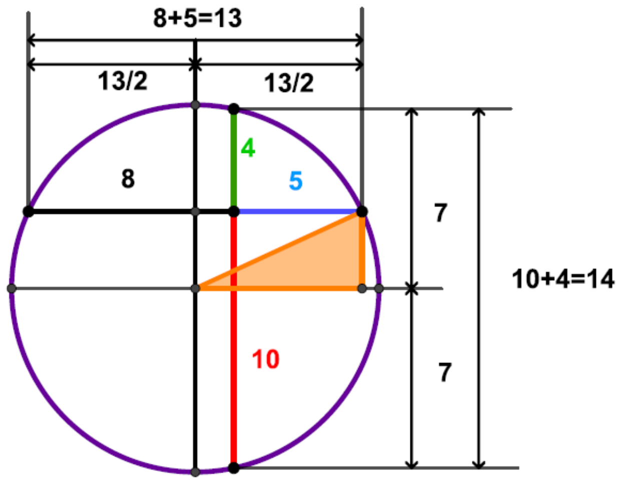
$$8^2 + 4^2 + 5^2 + 10^2 = 4r^2 \Rightarrow r = \frac{\sqrt{205}}{2} \quad \Rightarrow \quad \pi r^2 = \frac{205\pi}{4}$$

Solution 2 (without resorting to Faure's theorem): Applying the string theorem:

$$5y = 4 \cdot 10 \quad \Rightarrow \quad y = 8 \quad \Rightarrow \quad y^2 = x = 64$$

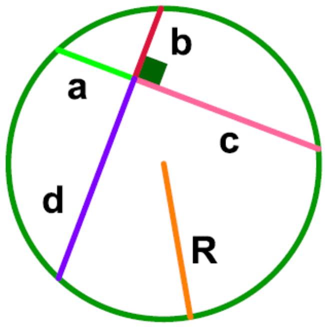
Consider the perpendiculars to the two chords through their midpoints. These two perpendiculars will intersect at the center of the circle. The orange right triangle will thus be formed, with legs 13/2 and (7 – 4 =) 3 and hypotenuse, the radius of the circle. Applying the Pythagorean theorem we get:

$$r = \sqrt{3^2 + \left(\frac{13}{2}\right)^2} = \frac{\sqrt{205}}{2} \Rightarrow \pi r^2 = \frac{205\pi}{4}$$

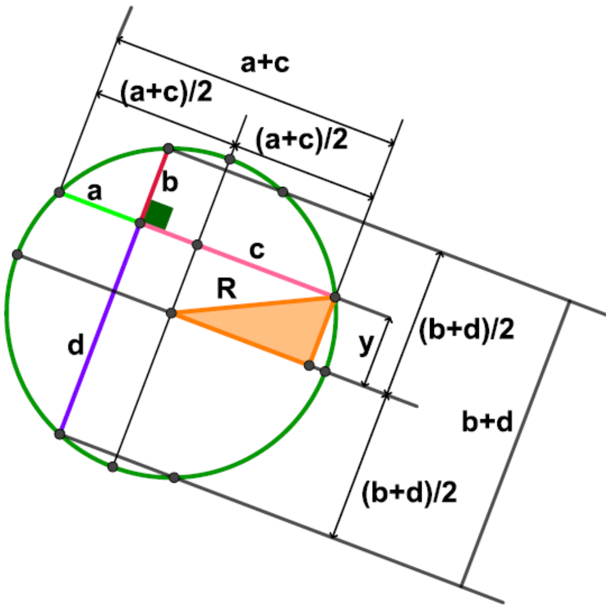


September 27: (Faure's theorem) In a circle of radius R and two perpendicular chords:

$$\mathbf{a^2 + b^2 + c^2 + d^2 = 4R^2}$$



Solution:



We draw the perpendiculars to the two chords through their midpoints. The intersection of these perpendiculars is the center of the circle. The orange triangle will be generated, with legs:

$$\frac{a+c}{2}$$

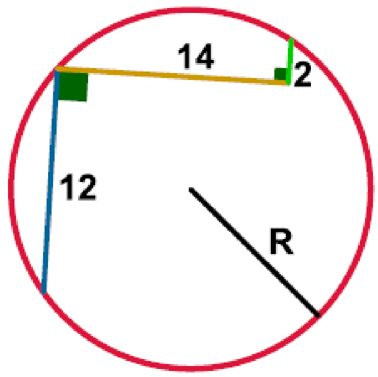
$$y = \frac{b+d}{2} - b = \frac{d-b}{2}$$

and hypotenuse R. Since the chords are perpendicular, the triangle is right and applying the Pythagorean theorem:

$$R^2 = \frac{(a+c)^2}{4} + \frac{(d-b)^2}{4}$$
$$= \frac{a^2+b^2+c^2+d^2}{4} - 2(ac-bd)$$
$$= \frac{a^2+b^2+c^2+d^2}{4} - 2 \cdot 0$$

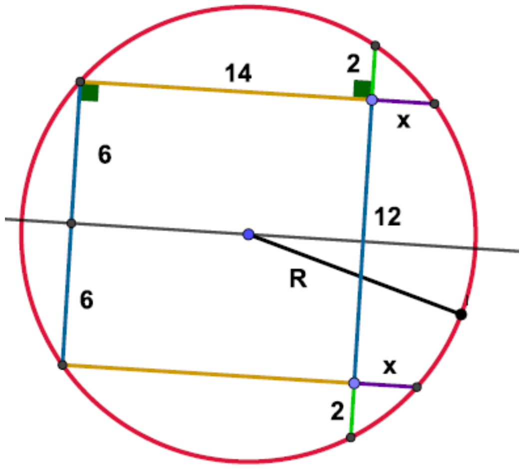
(since a c = b d, by the string theorem). From where:

$$4R^2 = a^2 + b^2 + c^2 + d^2$$



September 28: Find the radius of the circle.

Solution:



Consider the line perpendicular to the chord that measures 12 through its midpoint, which will pass through the center of the circle. We find the symmetrical points of the ends of the given segments with respect to this perpendicular and we will then have the attached figure.

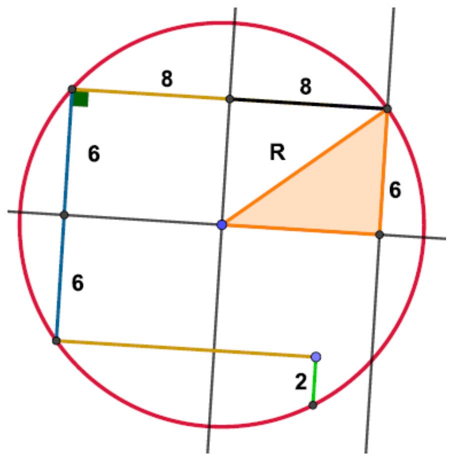
We apply the string theorem and get:

$$14x = (12 + 2) \cdot x \;\; \Rightarrow \;\; x = 2$$

And now to finish:

1.- or we apply Faure's theorem (taking advantage of the fact that the chords are perpendicular):

$$4R^2 = 2^2 + 2^2 + 14^2 + (12 + 2)^2 \;\; \Rightarrow \;\; R = 10$$

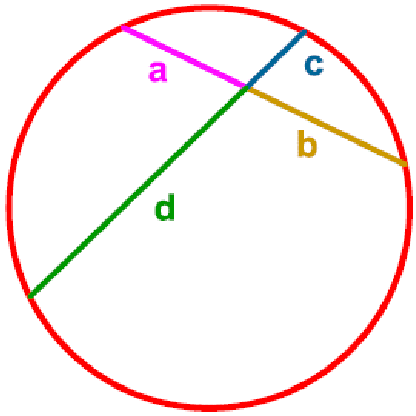


2.- Or, we retrace the perpendicular to the chord measuring (14 + 2 =) 16 through its midpoint that will intersect the other perpendicular at the center of the circle. Then the orange triangle appears, which is a rectangle (since the segments provided at the beginning are perpendicular). Applying the Pythagorean theorem to this orange triangle, we have:

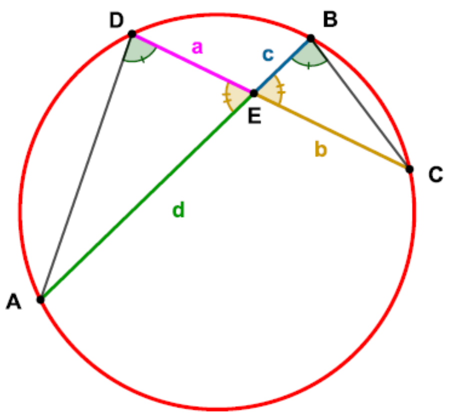
$$R = \sqrt{8^2 + 6^2} = 10$$

September 29: STRING THEOREM:

a·b = c·d



Solution:

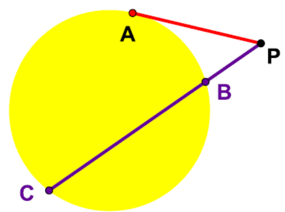


Triangles ΔADE and ΔCBE are similar, since the angles at E are opposite at the vertex and the angles at D and B subtend the same arc (arc AC). So:

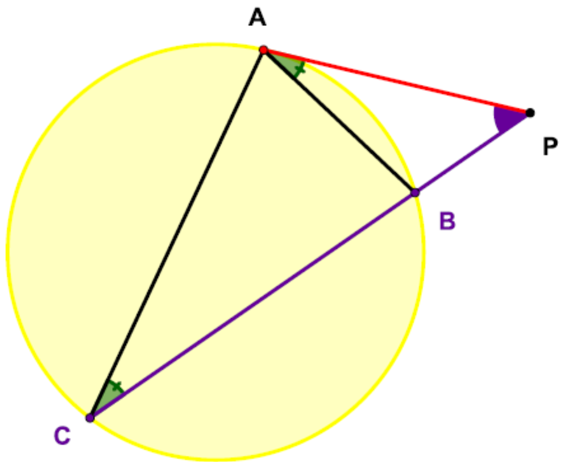
$$\frac{a}{c} = \frac{d}{b} \quad \Leftrightarrow \quad ab = cd$$

September 30: POWER OF A POINT: Circle, secant and tangent. Prove that:

$$PA^2 = PB \cdot PC$$



Solution:



Let us consider the triangles ▯CAP and ▯ABP. These two triangles are similar because they have in common the angle at P and the angle at A and the angle at C (which subtends the arc AB) are equal (If C approaches A by the circumference, the lines that pass through C and A tend to the tangent AP and the segment CB tends to the segment AB). Then we will have:

$$\frac{AP}{PC} = \frac{PB}{AP} \quad \Leftrightarrow \quad AP^2 = PC \cdot PB$$