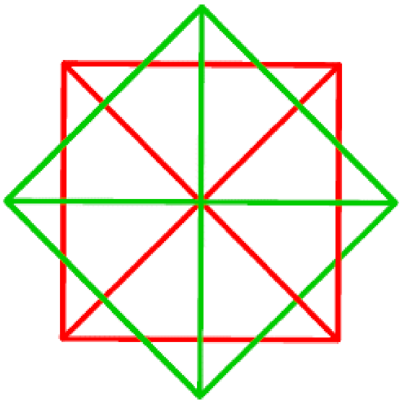
SOLUTIONS MARCH 2022

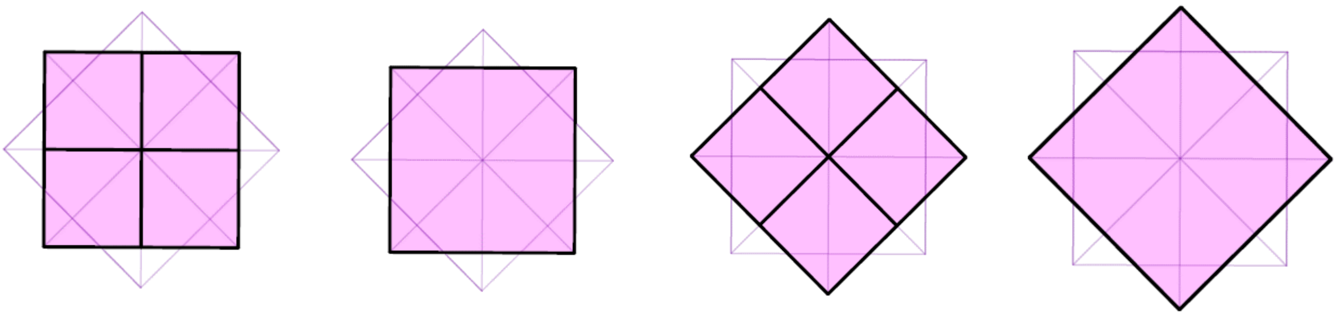
PROBLEMS FOR FIRST AND SECOND OF ESO. AUTHORS: Collective "CONCURSO DE PRIMAVERA"

http://www.concursoprimavera.es/#concurso.

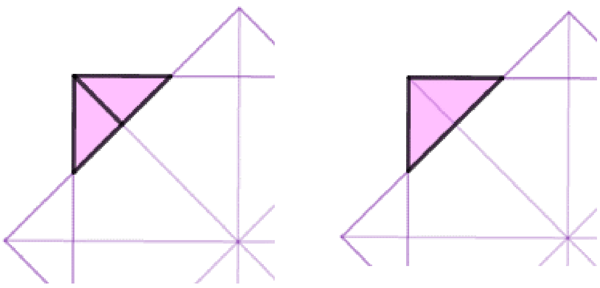
March 1: How many squares are there in the figure? And triangles?



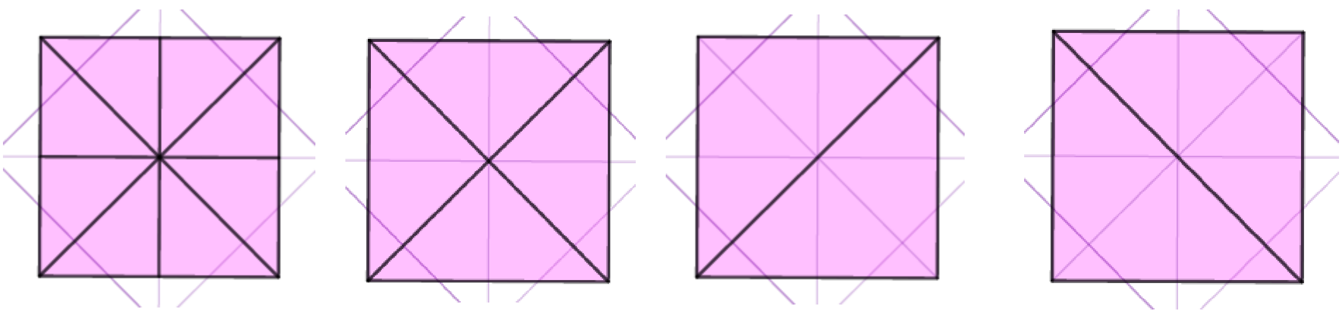
Solution: For the squares we have (4 + 1 + 4 + 1 =) 10:



For the triangles, we have:



Each point generates three triangles. Since there are a total of eight points, they total (8·3 =) 24 triangles



Each large square generates (8 + 4 + 2 + 2 =) 16 triangles. Since there are two large squares, we total (16·2 =) 32 triangles.

In total (24 + 32 =) 56 triangles

March 2: How many three-digit palindromic numbers are multiples of three? And of eleven?

Solution: Let xyx be the multiple of three. Then:

$$2x + y = \Sigma \in \{3, 6, 9, 12, 15, 18, 21, 24, 27\} \text{ with } x, y \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \; x \neq 0$$

x	y = multiple – 2x	total	palindromes
1	1, 4, 7	3	111, 141, 171
2	2, 5, 8	3	222, 252, 282
3	0, 3, 6, 9	4	303, 333, 363, 393
4	1, 4, 7	3	414, 444, 474
5	2, 5, 8	3	525, 555, 585
6	0, 3, 6, 9	4	606, 636. 666, 696
7	1, 4, 7	3	717, 747, 777
8	2, 5, 8	3	828, 858, 888
9	0, 3, 6, 9	4	909, 939, 969, 999

Then there are 30 three-digit palindromes that are multiples of three.

For palindromic numbers with three digits that are multiples of eleven: xyx, it must be fulfilled:

$$x, y \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \; x \neq 0 \text{ with } 2x - y = 0 \text{ o } 2x - y = 11$$

2x – y = 11

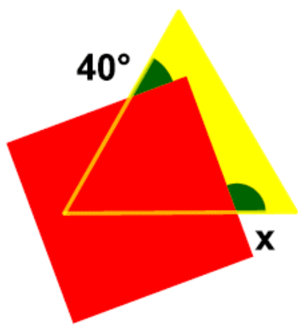
x	y	xyx
6	1	616
7	3	737
8	5	858
9	7	979

2x – y = 0

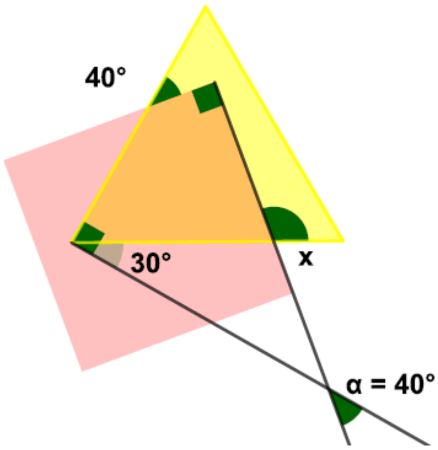
x	y	xyx
1	2	121
2	4	242
3	6	363
4	8	484

Then there are eight three-digit palindromes that are multiples of eleven.

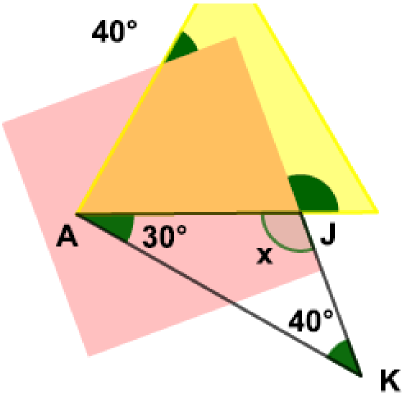
March 3: In the figure we have a square and an equilateral triangle. Find x



Solution:



In the attached figure α = 40° since 40° and α are angles with perpendicular sides.



Let us consider the triangle ΔAJK. In it we will have:

$$x = 180° - (30° + 40°) = 110°$$

March 4-5: Mr. Twisted has invented this game: He gives you a number, if it's even you multiply it by two and add one, if it's odd you multiply it by three and add one. If after applying the rule to the number that Mr. Twisted has given you and twice in a row to each of the numbers that you get, you reach 208, what number did Mr. Twisted give you?

Solution: We will use the "reverse" technique, following the steps in the statement backwards.

First, we subtract 1 from the final result (208) and divide it by 3, to obtain an odd number.:

$$208 - 1 = 207; \quad \frac{207}{3} = 69$$

Second, from the result obtained (69) we subtract 1 and divide it by two, to obtain an even number:

$$69 - 1 = 68; \quad \frac{68}{2} = 34$$

Third, from the result obtained (34) we subtract 1 and divide it by 3, to obtain an odd number:

$$34 - 1 = 33; \quad \frac{33}{3} = 11$$

Mr. Twisted gave us number 11.

March 7-8: Grandpa Gerardo has distributed his coin collection among his six grandchildren. He gave Carlos half of what he had. He gave Ferrán half of the ones he had left. He gave Dani half of what he had left and so he continued first with Laia, then with Aitana and finally with Clara and he kept three coins. How many coins did he have at the beginning, and how many did he give to each grandchild?

Solution: Let x be the number of coins Gerardo had. Then:

$\frac{x}{2}$ are the coins that Carles receives (remaining $\frac{x}{2}$ coins)

$\frac{x}{4}$ are the coins that Ferran receives (remaining $\frac{x}{4}$ coins)

$\frac{x}{8}$ are the coins that Dani receives (remaining $\frac{x}{8}$ coins)

$\frac{x}{16}$ are the coins that Laia receives (remaining $\frac{x}{16}$ coins)

$\frac{x}{32}$ are the coins that Aitana receives (remaining $\frac{x}{32}$ coins)

$\frac{x}{64}$ are the coins that Clara receives (remaining $\frac{x}{64}$ coins)

3 are the coins that Gerardo keeps.

Then we have the equation:

$$\frac{x}{2}+\frac{x}{4}+\frac{x}{8}+\frac{x}{16}+\frac{x}{32}+\frac{x}{64}+3=x \;\Rightarrow 64x=32x+16x+8x+4x+2x+x+192 \;\Rightarrow\; x=192$$

Corresponding: 96 coins for Carles, 48 coins for Ferran, 24 coins for Dani, 12 coins for Laia, 6 coins for Aitana, 3 coins for Clara and 3 coins for Gerardo.

Alternatively, with the "reverse" technique we have:

Gerardo keeps 3 coins, Clara receives 3 coins, Aitana receives 6 coins, Laia receives 12 coins, Dani receives 24 coins, Ferran receives 48 coins, and Carles receives 96 coins. Totalize (3 +3 + 6 + 12 + 24 + 48 + 96 =) 192 coins.

March 9: The product of three different natural numbers is 30. What are the possible values of the sum of the three natural numbers?

Solution: Since 30 = 2·3·5, there are only four possibilities:

30 = 2·3·5 and the sum of factors is S (= 2 + 3 + 5) = 10

30 = 1·6·5 and the sum of factors is S (= 1 + 6 + 5) = 12

30 = 1·2·15 and the sum of factors is S (= 1 + 2 + 15) = 18

30 = 1·3·10 and the sum of factors is S (= 1 + 3 + 10) = 14

March 10: Delete three digits in the number above and in the number below so that the result of the new subtraction is the smallest possible

$$\begin{array}{r} 7\;9\;5\;1\;6\;3 \\ -\;4\;9\;6\;7\;1\;8 \\ \hline 2\;9\;8\;4\;4\;5 \end{array}$$

Solution: If we admit that we can work with negative numbers, we have to generate the smallest possible number in the minuend: 163, and the largest possible number in the subtrahend: 978, leaving:

	7	9	5	**1**	**6**	**3**			**1**	**6**	**3**
−	4	**9**	6	**7**	1	**8**	⇒	**−**	**9**	**7**	**8**
								−	**8**	**1**	**5**

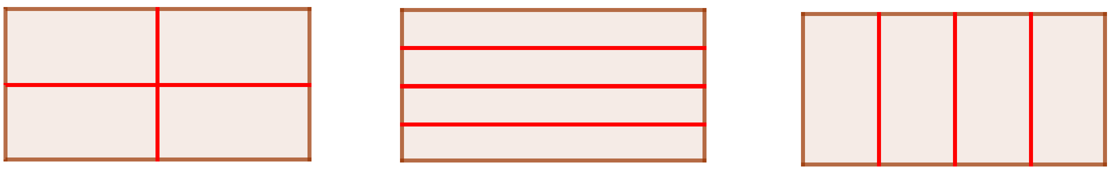
If we admit that it is only valid to work with non-negative integers, then we have to generate two numbers as close to each other as possible, the second smaller than the first, if possible with the first two figures equal and the third as close as possible.:

	7	**9**	5	1	**6**	**3**			**9**	**6**	**3**
−	4	**9**	**6**	7	**1**	8	⇒	**−**	**9**	**6**	**1**
									0	**0**	**2**

<u>March 11-18:</u> Dani has a sheet of dimensions 40 cmx20 cm. With three cuts, she divides this sheet into four equal rectangles. Each one of those rectangles divides them into four other equal ones, with the same type of cuts. This last operation she repeats twice. What is the perimeter of all the rectangles that are obtained at the end?

<u>Solution:</u> The operation of making the three cuts is repeated a total of four times. Three ways of making the cuts are possible.

A B C



Let's look at each of the three possibilities.

A. The first rectangle has dimensions: 40x20. The second rectangles have dimensions: 20x10. The third rectangles have dimensions: 10x5. The fourth rectangles have dimensions: 5x2.5. The fifth rectangles have dimensions: 2.5x1.25. The perimeter of each of these rectangles is: ((2.5 +1.25)·2 =) 7.5 cm. The sum of all the perimeters is: (4 4 4 4 7.5 =) 1920 cm.
B. The first rectangle has dimensions: 40x20. The second rectangles have dimensions: 40x5. The third rectangles have dimensions: 40x1.25. The fourth rectangles have dimensions: 40x0.3125. The fifth rectangles have dimensions: 40x0.078125. The perimeter of each of these rectangles is: ((40 +0.078125) 2 =) 80.15625 cm. The sum of all the perimeters is: (4 4 4 4 80.15625 =) 20520 cm.
C. The first rectangle has dimensions: 40x20. The second rectangles have dimensions: 5x20. The third rectangles have dimensions: 1.25x20. The fourth rectangles have dimensions: 0.3125x20. The fifth rectangles have dimensions: 0.078125x20. The perimeter of each of these rectangles is: ((20 +0.078125) 2 =) 40.15625 cm. The sum of all the perimeters is: (4 4 4 4 40.15625 =) 10280 cm.

March 12: Dani and other partners have formed the AVANT rock. At parties, each member has invited as many people as fellow members of the club. If it is known that there will be more than 66 attendees and less than 99, how many people will attend the event?

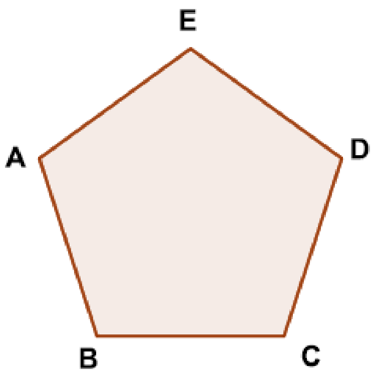
Solution: Let x be the number of partners. For each partner there are (x – 1) other partners and therefore (x – 1) guests. The total number of people invited is x·(x – 1) to which the founding members must be added. The total number of people attending is x + x·(x – 1) = x^2. Since the number of attendees is between 66 and 99, we have:

$$66 < x^2 < 99 \;\Rightarrow\; \sqrt{66} = 8{,}12\cdots < x < 9{,}94\cdots = \sqrt{99} \;\Rightarrow\; x = 9 \;\Rightarrow\; x^2 = 81$$

A total of 81 people will attend the event.

March 14: How many triangles can we form that have their vertices at the vertices of a regular pentagon? And in a regular hexagon?

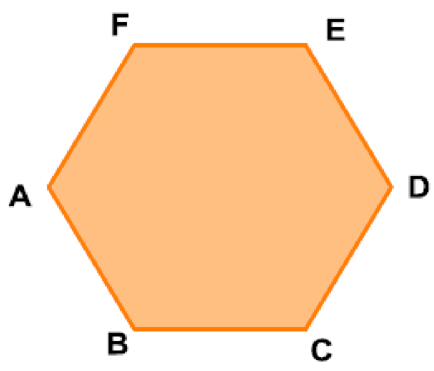
Solution: For the regular pentagon we have:



vertex 1	vertex 2	vertex 3	triangle
A	B	C	ABC
		D	ABD
		E	ABE
	C	D	ACD
		E	ACE
	D	E	ADE
B	C	D	BCD
		E	BCE
	D	E	BDE
C	D	E	CDE

10 triangles

For the regular hexagon we have:



vertex 1	vertex 2	vertex 3	triangle
A	B	C	ABC
		D	ABD
		E	ABE
		F	ABF
	C	D	ACD
		E	ACE
		F	ACF
	D	E	ADE
		F	ADF
	E	F	AEF
B	C	D	BCD
		E	BCE
		F	BCF
	D	E	BDE
		F	BDF
	E	F	BEF
C	D	E	CDE
		F	CDF
	E	F	CEF
D	E	F	DEF

20 triangles

March 15: Using the digits 8, 0, 7, 2, 6, 2, 5, 4 only once each, we must generate four two-digit numbers less than 53 so that no two of them are consecutive. Which are?

Solution: Note that the 0, 6, 7, and 8 must occupy the ones positions. The 5 can only occupy the tens position with the 0 in the units and, so that there are no consecutive numbers, the 2 will have to go with the 6 and 8. Thus, the numbers are: 26, 28, 47 and 50.

March 16-23:

			21
			60
			288
112	72	45	

Place all the natural numbers from 1 to 9 without repeating any in the attached matrix, taking into account that the outer numbers indicate the product of the numbers located in the row or column

Solution:

7			$3\cdot7$
		5	$2^2\cdot3\cdot5$
			$2^5\cdot3^2$
$2^4\cdot7$	$2^3\cdot3^2$	$5\cdot3^2$	

We obtain the factorial decomposition of the row and column products. The cells where the 5 and 7 are easily located: they are cells where the factors 7 and 5 appear in the row and column products.

We now look at the third column. It remains to place a nine (because two threes are not possible in that column). Therefore we have what is described in the following diagram

7		1	$3\cdot7$
		5	$2^2\cdot3\cdot5$
		9	$2^5\cdot3^2$
$2^4\cdot7$	$2^3\cdot3^2$	$5\cdot3^2$	

Now in the first row appears the 3 in second position

7	3	1	$3\cdot7$
		5	$2^2\cdot3\cdot5$
		9	$2^5\cdot3^2$
$2^4\cdot7$	$2^3\cdot3^2$	$5\cdot3^2$	

Now in the second row a two and a six must appear (because a 3 cannot appear). Since only powers of two should appear in the first column, we place the 2 there

7	3	1	$3\cdot7$
2	6	5	$2^2\cdot3\cdot5$
		9	$2^5\cdot3^2$
$2^4\cdot7$	$2^3\cdot3^2$	$5\cdot3^2$	

In the first column an 8 should appear and therefore in the second column a 4

7	3	1	$3\cdot7$
2	6	5	$2^2\cdot3\cdot5$
8	4	9	$2^5\cdot3^2$
$2^4\cdot7$	$2^3\cdot3^2$	$5\cdot3^2$	

March 17: A meter is a billion nanometres. To calculate the thickness of a sheet of paper, Lucia has observed that ten sheets measure one millimetre. How many nanometres is the thickness of a sheet?

Solution: From the statement we have 1m = $1.000_1000.000$ nm. So:

$$1\text{mm} = \frac{1}{1000}\ \text{m} = \frac{1000_1000.000}{1000}\ \text{nm} = 1_1000.000\ \text{nm}$$

Therefore, since the thickness of a sheet is one tenth of a millimetre:

$$\frac{1}{10}\ \text{mm} = \frac{1_{1}000.000}{10}\ \text{nm} = 100.000\ \text{nm}$$

The thickness of a sheet is 100,000 nanometres.

March 19: A, B, C, D and E represent different digits. If the product on the right is well done, calculate the value of each letter

1	**A**	**B**	**C**	**D**	**E**
				x	**3**
A	**B**	**C**	**D**	**E**	**1**

Solution 1 (@asitnof): Let N be the number formed by the digits A, B, C, D and E (and in that order). We will have, if the operation is well done, that:

$$(10^5 + \text{N}) \cdot 3 = 10\text{N} + 1 \ \Rightarrow\ 300.000 + 3\text{N} = 10\text{N} + 1 \ \Rightarrow 7\text{N} = 299.999 \ \Rightarrow \text{N} = \frac{299.999}{7} = 42857$$

Then A = 4, B = 2, C = 8, D = 5 y E = 7.

Solution 2:

1	A	B	C	D	E
				x	3
A	B	C	D	E	**1**

We have that 3 E must be 1 or 11 or 21. Reviewing the multiplication table of 3 we have that the only possible result is 3 E = 21, that is E = 7

				2	
1	A	B	C	D	**7**
				x	3
A	B	C	D	**7**	1

We have that 3 D + 2 must be 7 or 17 or 27. That is, 3 D must be 5, 15 or 25. Reviewing the multiplication table of 3 we have that the only possible result is 3 D = 15, that is D = 5

			1	**2**	
1	A	B	C	**5**	**7**
				x	3
A	B	C	**5**	**7**	**1**

We have that 3 C + 1 must be 5 or 15 or 25. That is, 3 C must be 4, 14 or 24. Reviewing the multiplication table of 3 we have that the only possible result is 3 C = 24, that is C=8

		2	**1**	**2**	
1	A	B	**8**	**5**	**7**
				x	3
A	B	**8**	**5**	**7**	**1**

We have that 3 B + 2 must be 8 or 18 or 28. That is, 3 B must be 6, 16 or 26. Reviewing the multiplication table of 3 we have that the only possible result is 3 B = 6, that is B = 2

		2	**1**	**2**	
1	A	**2**	**8**	**5**	**7**
				x	3
A	**2**	**8**	**5**	**7**	**1**

We have that 3 A must be 2 or 12 or 22. Reviewing the multiplication table of 3 we have that the only possible result is 3 A = 12, that is A = 4

1		**2**	**1**	**2**	
1	**4**	**2**	**8**	**5**	**7**
				x	3
4	**2**	**8**	**5**	**7**	**1**

March 21: Find the possible values of A and B if ¾ of 2/5 of A is equal to 2/3 of 3/5 of B

Solution: We have:

$$\frac{3}{4}\cdot\frac{2}{5}\cdot A=\frac{2}{3}\cdot\frac{3}{5}\cdot B \quad\Rightarrow\quad 3A=4B \ \ (1)$$

Due to the uniqueness of the factorial decomposition into prime factors, we have that A must be a multiple of 4 and B must be a multiple of 3. That is, it must be: A = 4·P and B = 3·K. So by substituting into (1), we have:

$$3\cdot 4\cdot P=4\cdot 3\cdot K \quad\Rightarrow\quad P=K$$

Then the solutions of the given equation are:

$$A=4K \text{ y } B=3K \text{ with } K\in\{1,2,3,4,\cdots\cdots\}$$

March 22: Order from largest to smallest: $\mathbf{11^{525}}$, $\mathbf{1317^{175}}$, $\mathbf{37^{350}}$

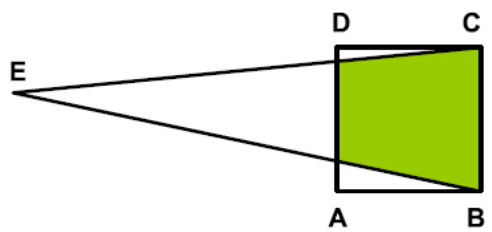
Solution: We will have:

$$\left.\begin{array}{r} 11^{525}=(11^3)^{175}=1331^{175} \\ 1317^{175} \\ 37^{350}=(37^2)^{175}=1369^{175} \end{array}\right\}$$

Like: 1369 > 1331 > 1317, we will have:

$$1369^{175}=\mathbf{37^{350}}>1331^{175}=\mathbf{11^{525}}>\mathbf{1317^{175}}$$

March 24-25: The area of square ABCD is 16 cm^2 and that of triangle ΔBCE is 32 cm^2. Find the area of the shaded trapezoid.



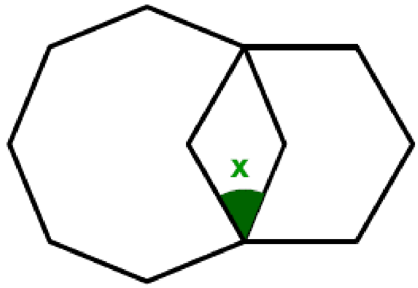
Solution: We know that the side of the square ABCD measures 4 cm and, therefore, the height of the triangle ΔBCE (which has an area of 32 cm^2) must measure 16 cm. To calculate the area of the trapezoid we need to know the length of its smaller base: b. Note that the white triangle on the left is similar to the triangle ΔBCE (because they are in Thales position), so the ratio between their heights is equal to the ratio between their bases:

$$\frac{16-4}{16}=\frac{b}{4} \quad\Rightarrow\quad b=\frac{4\cdot 12}{16}=3\text{ cm}$$

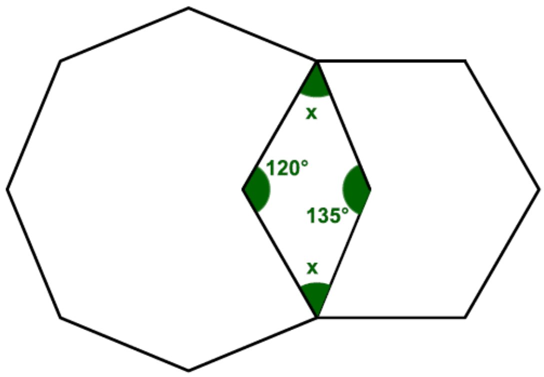
The area of the trapezoid is:

$$\frac{4+3}{2}\cdot 4=14\text{ cm}^2$$

March 26: In the figure there is a regular hexagon and a regular octagon. Find the measure of angle x



Solution:



The angle between two consecutive edges of a regular hexagon is:

$$180° - \frac{360°}{6} = 120°$$

The angle between two consecutive edges of a regular octagon is:

$$180° - \frac{360°}{8} = 135°$$

Since the quadrilateral generated by the hexagon and the octagon generates two triangles, their interior angles must add up to (2·180° =) 360°. From here:

$$2x + 120° + 135° = 360° \quad \Rightarrow x = \frac{105°}{2} = 52°\,30'$$

March 28-29: Aitana has written all the natural numbers she knows how to write on a sheet of paper. Laia has erased those that, according to her, are primes and has added them, obtaining 230. The older brother, Dani, congratulates Aitana because she has not forgotten any number and tells Laia that she has added a number that is not prime. Up to what number has Aitana written? What number has Laia considered a prime number and she is not?

Solution: We add the initial prime numbers and stop the first time we exceed 230.

2	3	5	7	11	13	17	19	23	29	31	37	41	....
	5	10	17	28	41	58	77	100	129	160	197	238	....

Then if Laia has made a mistake when considering a composite number as prime, Laia has considered the number (230 – 197 =) 33 as prime and Aitana has written down up to the number 37 or 38 or 39 or 40.

March 30: Dani collects geometric figures. Half of the ones he has are triangles, a third of the rest are circles, and a quarter of the rest are trapezoids. If he has 20 trapezoids, how many triangles and circles does he have?

Solution: Let x = number of geometric pieces that Dani has

$$\frac{x}{2} = \text{number of triangles. Remain } \left(x - \frac{x}{2} =\right) \frac{x}{2} \text{ geometric figures}$$

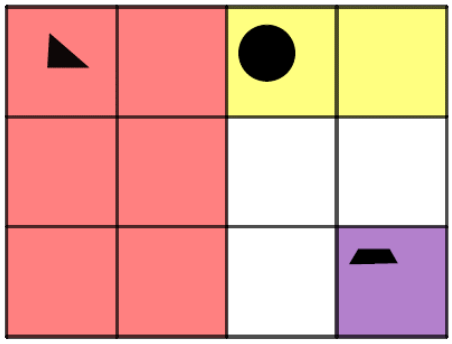
$$\frac{1}{3} \cdot \frac{x}{2} = \frac{x}{6} = \text{ number of circles. Remain } \left(\frac{x}{2} - \frac{x}{6} =\right) \frac{x}{3} \text{ geometric figures}$$

$$\frac{1}{4}\cdot\frac{x}{3}=\frac{x}{12}=\text{ number of trapezoids }=20\ \Rightarrow\ x=12\cdot 20=240.$$

$$\frac{x}{2}=\frac{240}{2}=120\text{ triangles and }\frac{x}{6}=\frac{240}{6}=40\text{ circles}$$

A graphical way of doing the problem is the one exposed to the side:

Each cell equals 20 figures. There are two cells of circles, one cell of trapezoids, and six cells of triangles.



<u>March 31:</u> There are 200 people in a movie theatre. Of them 130 are women. Also, there are 90 people who wear glasses. If half of the men wear glasses, how many women don't?

<u>Solution:</u> We build the following contingency table, in which the data of the problem statement appear (M = women; H = men G = wear glasses, no G = do not wear glasses):

	M	H	
G			**90**
no G			
	130		**200**

There will be (200 – 130 =) 70 men, of which half (i.e. 35) wear glasses

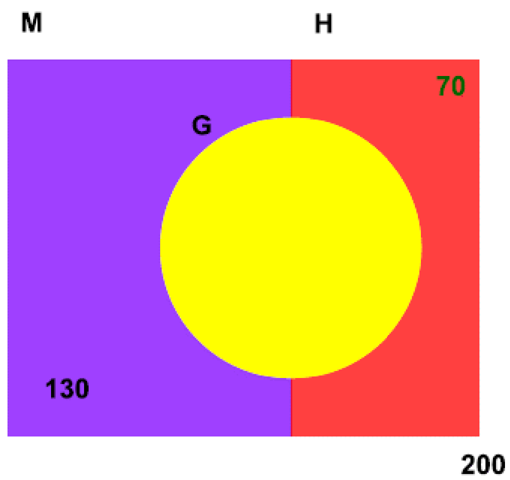
	M	H	
G		**35**	**90**
no G			
	130	**70**	**200**

Women who wear glasses will be (90 – 35 =) 55, and therefore women who do not wear glasses will be (130 – 55 =) 75

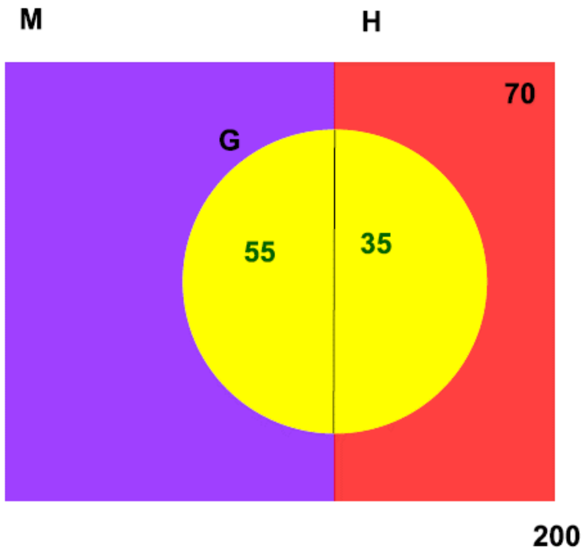
	M	H	
G	**55**	**35**	**90**
no G	**75**		
	130	**70**	**200**

	M	H	
G	**55**	**35**	**90**
no G	**75**	**35**	**110**
	130	**70**	**200**

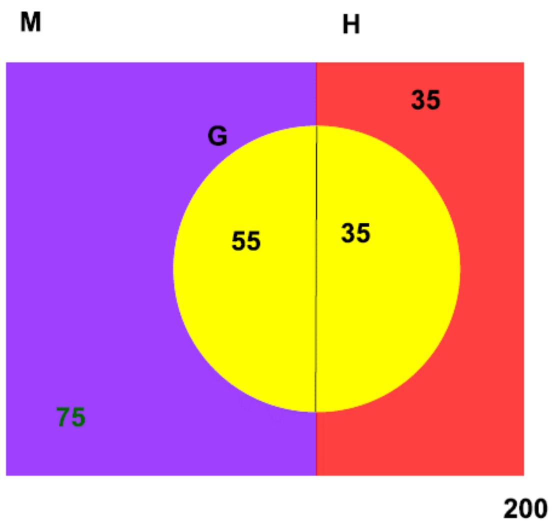
Another way to solve the problem is to resort to Venn diagrams.:



Of 200 people, 130 are women, therefore there are (200 – 130 =) 70 men



Of the 70 men, half (35) wear glasses. Since 90 are people who wear glasses (90 – 35 =) 55 are women who wear glasses



Since there are 130 women and 55 of them wear glasses, we have that there are (130 – 55 =) 75 women who do not wear glasses