JUNE

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY	U



1 Let ABCDEFG be a regular heptagon with side 1. Prove that:

$$\frac{1}{AC}+\frac{1}{AD}=1$$

2 Five positive integers (not necessarily different) are written on the board and all possible sums of pairs of these numbers are calculated. The only results obtained are 31, 38 and 45 (some of them, several times). What are those 5 numbers?

3 Let S be a set of n elements. Let $p_n(k)$ the number of permutations of the elements of S that leave exactly k elements fixed. Show:

$$\sum_{k=0}^{n} k p_n(k) = n!$$

IMO, 1987, PROBLEM 1

4

5 If α, β and γ ∈ℝ are the angles of a non-right triangle, show that:

$$\tan(\alpha)+\tan(\beta)+\tan(\gamma) = \tan(\alpha)\cdot\tan(\beta)\cdot\tan(\gamma)$$

6 How many odd factors does 20! have?

7 Find all the polynomials P (x) and Q (x) with real coefficients that satisfy:

$$P(Q(x)) = (x-1)\cdot(x-2)\cdot(x-3)\cdot(x-4)$$

8 In each square of a mxn board there is a real number. It is allowed to change all the numbers in a row or column as many times as we want. Show that the sums of the elements in each row and each column can be made non-negative for any initial configuration.

ALL SOVIET UNION COMPETITION 1961. P 7

9 Let a_1, a_2, …. a non-constant PA of real numbers: Suppose there are integers that are prime to each other p, q > 1 for those who a_1^2, a_{p+1}^2 y a_{q+1}^2 are also elements of the same sequence. Prove that all the terms of the sequence are integers.

Indian National Mathematical Olympiad, 2016, problem 6

10 Let x, y and z reals distinct and distinct from 1 and also:

$$\frac{yz-x^2}{1-x}=\frac{xz-y^2}{1-y}$$

prove that both fractions are equal to x + y + z

Olimpiada Iberoamericana, 1985, problema 4

11

12 Consider a convex polygon of area A and perimeter P. Prove that there exists a circle of radius A/P contained inside the polygon

ALL SOVIET UNION COMPETITION 1966, P 7

13 Let n∈ℕ. Prove that the sum of all fractions 1/(pq) where p and q are relatively prime such that 1≤p<q≤n y p+q > n is 1/2

14 On a table there are 100 cards numbered from 1 to 100. 25 of them are chosen at random, what is the probability that the sum of the chosen numbers is even?

15 Prove that if x, y ∈]-1, 1[, then:

$$\left|\frac{x-y}{1-xy}\right| = \frac{|x|+|y|}{1+|xy|}$$

OME, fase local 2004, problema 7

16 Let us consider the set:

S = {1, ½, 1/3, ¼, ⋯⋯ , 1/1000}

We repeat the following process until there is only one element left in S: we choose two numbers x, y∈S and substitute the number x + y + xy. Prove that the last number does not depend on the numbers chosen at each step and calculate it.

17 Find all ways to express 2003 as the sum of the squares of two integers.

OME, fase local 2004, problema 9

18

19 Given a set M of 1985 positive integers, none of which has a prime divisor greater than 26, show that we can find 4 distinct elements of M whose geometric mean is an integer

IMO, 1985, P 4

20 We will say that a circumference is a separator of a set of 5 points in the plane if it passes through 3 of them and the other two, one is inside and the other is outside. Prove that every set of 5 points that does not contain 3 aligned points and 4 concyclic points has exactly 4 separators.

IMO, SHORTLIST GEOMETRY, 1999, P 2

21 Find the value of the real m so that the polynomial

$$x^4-\frac{3\sqrt{2}}{2}x^3+3x^2+mx+2$$

has two real roots, one inverse of the other

22 In a square ABCD a circle is drawn passing through the vertex A and through the midpoints of BC and CD. Determine if the length of the circumference is greater than the perimeter of the square.

23 Let P be an interior point of an equilateral triangle ΔABC such that PA = 5, PB = 7 and PC = 8. Find the length of one side of the ΔABC.

Olimpiada Iberoamericana, 1985, problema 2

24 Let P(x) be a polynomial with integer coefficients such that the equation P(x) = 7 has at least four integer solutions.

Prove that the equation P(x) = 14 has no integer solutions.

25

26 Given k∈ℕ, let A_k the subset of {k+1, k+2,..,2k} formed by the numbers that in base 2 have exactly three ones and is f(k) formed by the numbers that in base 2 have exactly three ones and is A_k. Prove that f (k) = m has at least one solution ∀m∈ℕ. Find the m∈ℕ for which the equation has a unique solution.

27 Let p and q be integers such that:

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots-\frac{1}{1318}+\frac{1}{1319}=\frac{p}{q}$$

Prove that 1979|p

IMO, 1979, PROBLEM 1

28 Consider a triangle whose sides are the sides of a regulars pentagon, hexagon and decagon inscribed in circles of radius 1. Prove that the triangle is a right triangle

29 Suppose that α and β are real that satisfy the equations:

$$\alpha^3-3\alpha^2+5\alpha-17=0$$
$$\beta^3-3\beta^2+5\beta+11=0$$

Calculate α + β

30 Let x, y and z three reals such that:

$$0<x<y<z<\frac{\pi}{2}$$

Prove that:

$$\frac{\pi}{2}+2\sin x\cos y+2\sin y\cos z > \sin 2x+\sin 2y+\sin 2z$$

Olimpiada Iberoamericana, 1989, problema 2